January 2011

Control charts are really a study in variation. What does it mean when a control chart indicates that a process is in statistical control? It means it is consistent and predictable - you can predict what the process will do in the near future. But it also means that the short-term variation is consistent with long-term variation. If you are using an Xbar-R chart, the short-term variation is given by the within-subgroup variation on the range chart. The long-term variation is given by the between subgroup variation on the Xbar chart. But what happens when the process is such that is not possible for the short-term variation to be consistent with the long-term variation?

This month's newsletter takes a look at how to address situations with control charts where the within-subgroup variation is significantly different from the between-subgroup variation. We start with a brief review of Xbar-R charts. We then take a look at what the Xbar-R control charts often looks like when you have situation where there is a significant difference between the within-subgroup variation the between-subgroup variation. You will see that the control limits on the Xbar chart are very tight and most of the points on that chart are beyond the control limits. We will then look at how to handle these types of situations by using three control charts.

In this issue:

Classical Xbar-R Control Charts

The classical Xbar-R control chart is designed to look at two types of variation:

- The range chart examines the variation within a subgroup
- The Xbar chart examines the variation between subgroups

Suppose you are making a product. At the start of each hour, you collect five samples from the production line and measure a quality characteristic, X. This is your subgroup. You take those five measurement results and calculate a range. The range is the maximum value - minimum value. It is a measure of the variation in the individual results. You plot those range values on the R chart - which monitors the variation within a subgroup. You also calculate the subgroup average - the average of the five measurement results. You plot the average on the Xbar chart - which monitors the variation between subgroup averages.

After you have enough points, you calculate the overall averages and the control limits. The control limits for the Xbar chart are given below.

$$UCLx = \bar{\bar{X}} + A_2\bar{R}$$

$$LCLx = \bar{\bar{X}} - A_2\bar{R}$$

The X with the two lines above ("X double bar") is the overall average; Rbar is the average range and A_2 is control chart constant that depends on subgroup size. The control limits for the R chart are given below.

$$UCLr = D_4\bar{R}$$

$$LCLr = D_3\bar{R}$$

D4 and D3 are control chart constants that depend on subgroup size. Please see our two-part newsletter series on Xbar-R control charts if you would like more information on Xbar-R control charts.

Note that the control limits for the Xbar chart are based on the average range (Rbar). If a process is in statistical control, it means that the within-subgroup variation (measured by the range chart) is consistent with the between-subgroup variation (measured by the Xbar chart).

But what happens if it is not reasonable to expect the within-subgroup variation to be consistent with the between-subgroup variation. For example, you might have a batch process and you take multiple samples from a batch and form a subgroup. In this case, the range is monitoring the within-batch variation and the Xbar chart is monitoring the between subgroup variation in subgroup averages. It might be that the average range is very small and almost all the points on the Xbar chart are out of control. How do you handle this situation?

Xbar-R Control Chart Example: Small Average Range

Consider the following example. You are running a batch reactor. Each batch takes about four hours to run. You measure the purity of the batch four times in the last hour to ensure that it has stabilized. You want to monitor the results using an Xbar-R chart. The data you have collected for the first 10 batches are given the table below. X_1, X_2, X_3 and X_4 are the four samples you pull from the batch in the last hour.

Subgroup	X_1	X_2	X_3	X_4	Xbar	R
1	98.4	98.6	98.3	98.7	98.5	0.4

2	97.5	97.6	98.0	97.6	97.7	0.5
3	98.8	98.9	98.4	98.7	98.7	0.5
4	99.1	99.3	99.4	99.2	99.3	0.3
5	97.8	98.0	98.2	98.0	98.0	0.4
6	98.3	98.5	98.5	98.5	98.5	0.2
7	98.9	99.0	98.6	99.0	98.9	0.4
8	97.5	97.7	97.6	97.9	97.7	0.4
9	99.3	99.3	99.2	99.4	99.3	0.2
10	98.5	98.7	98.7	98.3	98.6	0.4

The subgroup averages (Xbar) and range (R) have been included in the table as well. You then plot the Xbar values on the Xbar chart and the range values on the R chart. Then you calculate the overall averages and control limits using the equations given above for the Xbar-R chart. A summary of the calculations are given below.

$$\overline{R} = \frac{\sum R_i}{k} = \frac{R_1 + R_2 + \ldots R_k}{k} = \frac{3.7}{10} = 0.37$$

$$\overline{\overline{X}} = \frac{\sum \overline{X}_i}{k} = \frac{\overline{X}_1 + \overline{X}_2 + \ldots \overline{X}_k}{k} = \frac{985.0}{10} = 98.5$$

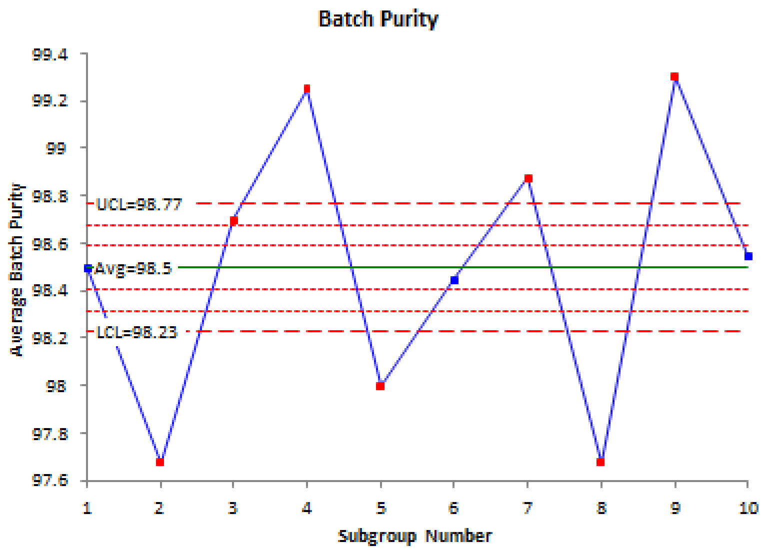
$$UCLr = D_4\overline{R} = (2.282)(0.37) = 0.84$$

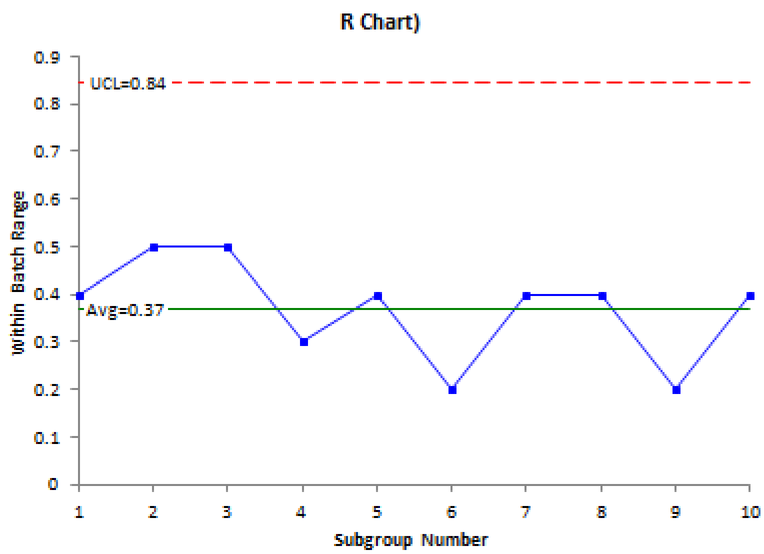
$$LCLr = D_3\overline{R} = \text{"None"}$$

$$UCLx = \overline{\overline{X}} + A_2\overline{R} = 98.5 + (0.729)(0.37) = 98.77$$

$$LCLx = \overline{\overline{X}} - A_2\overline{R} = 98.5 - (0.729)(0.37) = 98.23$$

The resulting control charts are shown below.





The range chart looks great. It is in statistical control. But the Xbar chart looks terrible - almost everything is out of control. It is easy to see what happened in this example. Look at the data. There is very little variation within a batch - an average range of 0.37. Now look at the batch averages. They vary a lot more. And since you use the average range in the calculation of the control limits for the Xbar chart, the control limits are very tight and everything looks like it is out of control. Plus, it is not reasonable to expect the within-subgroup variation to be consistent with the between-subgroup variation because of the process. So how do you handle this case?

Xbar-mR-R Chart: The Solution

There are really three sources of variation you want to monitor in this situation. One is the within-subgroup (batch) variation. This is given by the range chart above. So, we will keep that one. The issue is the Xbar chart. It doesn't help tell us anything about the process. To make it useful, we will exchange the Xbar chsart for two charts. We will use

the individuals chart (X-mR), but in this case, the individual values are really the subgroup averages. The mR chart will be the moving range between the subgroup averages.
The data are shown in the table below with addition of a mR column which is the moving range between consecutive subgroup averages. For example, the first mR listed is 0.8. That is the range between the first two subgroup averages: 98.5 and 99.7

Subgroup	X1	X2	X3	X4	Xbar	R	mR
1	98.4	98.6	98.3	98.7	98.5	0.4	
2	97.5	97.6	98.0	97.6	97.7	0.5	0.8
3	98.8	98.9	98.4	98.7	98.7	0.5	1.0
4	99.1	99.3	99.4	99.2	99.3	0.3	0.5
5	97.8	98.0	98.2	98.0	98.0	0.4	1.2
6	98.3	98.5	98.5	98.5	98.5	0.2	0.5
7	98.9	99.0	98.6	99.0	98.9	0.4	0.4
8	97.5	97.7	97.6	97.9	97.7	0.4	1.2
9	99.3	99.3	99.2	99.4	99.3	0.2	1.6
10	98.5	98.7	98.7	98.3	98.6	0.4	0.8

The calculations for the X-mR based on subgroup averages are shown below. Please see our newsletter for more information on individuals control charts.

$$\overline{R} = \frac{\sum R}{k-1} = \frac{8.1}{10-1} = 0.9$$

$$\overline{\overline{X}} = \frac{\sum \overline{X}}{k} = \frac{985.0}{10} = 98.5$$

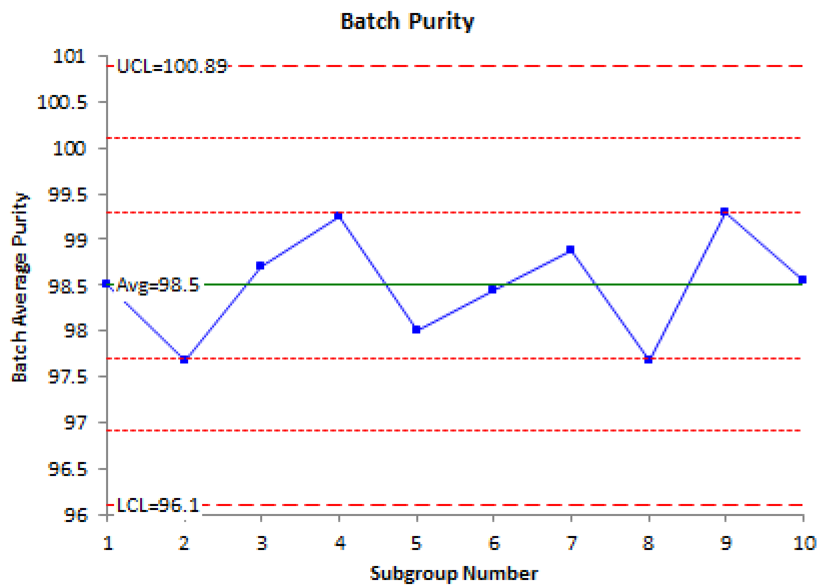
$$UCLr = 3.27\overline{R} = 3.27(0.9) = 2.94$$

$$LCLr = None$$

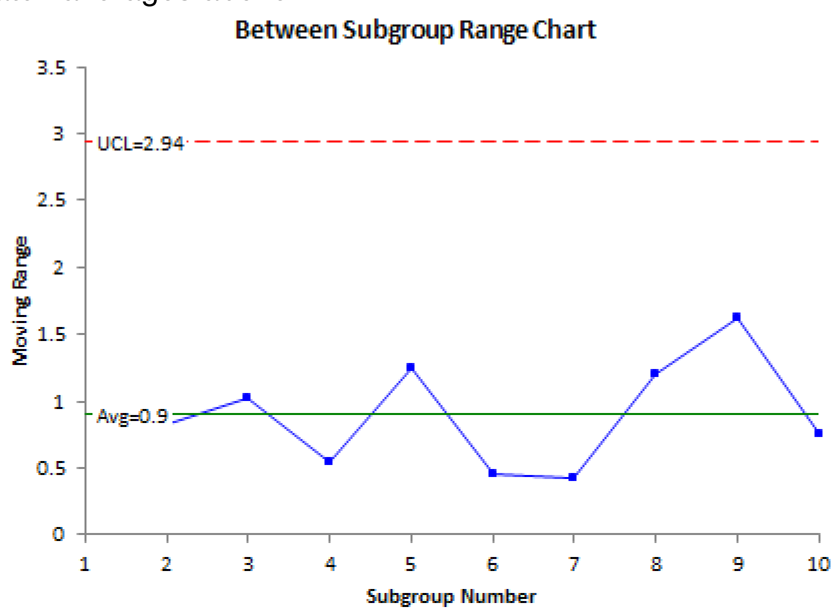
$$UCLx = \overline{X} + 2.66\overline{R} = 98.5 + 2.66(0.9) = 98.77$$

$$UCLx = \overline{X} - 2.66\overline{R} = 98.5 - 2.66(0.9) = 98.23$$

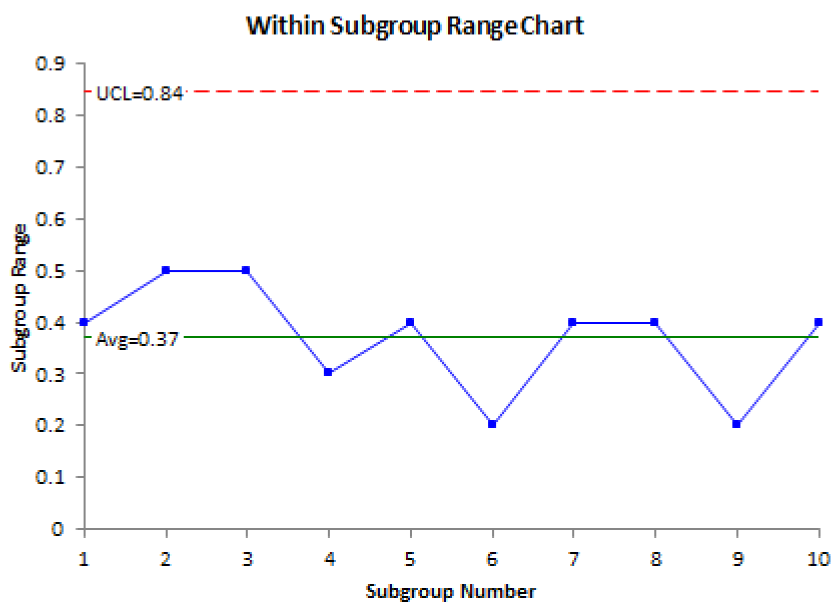
Note that the average range now used in the control chart equations is 0.9 compared with the 0.37 used in the classical Xbar-R calculations.
The Xbar Chart is shown below. Its control limits are based on the individuals control chart using each subgroup average as an individual value.



The moving range chart is shown below. This chart plots the moving range between subgroup averages. The average range on this chart was used to calculate the control limits on the batch averages above.



The third chart is the within-subgroup range chart. This is the same as the range chart shown before.



These three charts are in statistical control. They tell quite a different story than if the classical Xbar-R charts had been used. These three charts can be used to monitor processes where the within-subgroup variation is significantly different from the between-subgroup variation.

Summary

This newsletter has examined how to handle the situation when the within-subgroup variation is naturally different from the between-subgroup variation. In this case, the classical Xbar-R chart gives an Xbar chart where most of the points are out of statistical control. The approach described in this newsletter uses three control charts to handle the problem: an Xbar chart where the subgroup averages are treated as individual values and the limits are based on the moving range between the subgroup averages; the between subgroup range chart which is that moving range between the subgroup averages; and the within-subgroup range chart, which is the normal range chart for the Xbar-R chart.

October 2006

In this issue:

As promised in last month's e-zine, this is the first of two e-zines you will receive this month. This one covers individuals control charts. The next one will be the first of a four-part series on the teachings of Dr. W. Edwards Deming.

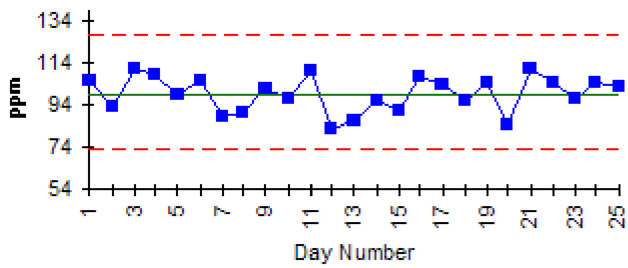
Suppose your process generates data on a very limited frequency. Maybe you only get data once a day, once a week, or once every two weeks. How can we apply control charts to these types of data? If we wait to get several data points to form a subgroup, we won't be able to plot a point very often. Perhaps the test method used to analyze the process is very expensive to run or takes a long time. How can we handle this type of situation? In these instances, individuals control charts are useful. This type of chart is useful when you have only one data point at a time to represent a given situation. The individuals control chart is introduced in this e-zine.

Introduction

The individuals control chart is a type of control chart that can be used with variables data. Like most other variables control charts, it is actually two charts. One chart is for the individual sample result (X). The other chart is for the moving range (R) between successive individual samples. The individuals chart is very useful for monitoring processes where data are not available on a frequent basis. The individuals control chart examines variation in individual sample results over time. While rational subgrouping does not apply, thought must be given to when the results will be measured. If the process is in statistical control, the average on the individuals chart is our estimate of the population average. The average range will be used to estimate the population standard deviation.

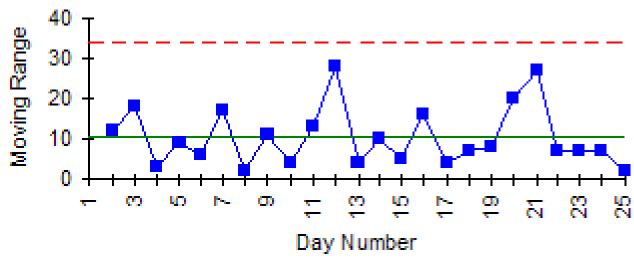
The individuals control chart is a method of looking at variation. One source of variation is the variation in the individual sample results. This represents "long-term" variation in the process. The second source of variation is the variation in the ranges between successive samples. This represents "short-term" variation.

X Chart



The figure in this section is an example of an X chart for individual results. In this example, a sample is pulled once a day from a given process stream and tested for a certain contaminant. Since data are not obtained very frequently, an individuals control chart was selected. On the first day the test method indicated that there were 105 ppm of the contaminant in the product stream. On the second day, the sample result was 93 ppm. The overall process average has been calculated and plotted as a solid line. The upper and lower control limits have also been calculated and plotted as dashed lines.

Moving Range Chart



The figure in this section is the moving range chart that goes with the X chart. This chart represents the range between successive data points. This range is often called a moving range. The range value plotted for the second day is simply the range between day 1 and day 2 (largest minus smallest). This range is 105 - 93 = 12. The average range has been calculated and plotted as a solid line. The upper control limit has also been calculated and plotted as a dashed line. There is no lower control limit on the range chart for an individuals chart.

Statistical Control

The charts above are in statistical control. What does it mean when the individuals control chart is in statistical control? It means that the individual sample results are consistent over time, i.e., they are not significantly different from the process average. It also means that the difference between successive sample results is consistent over time. We can predict what the process will make in the near future. For more information on interpreting control charts, please see our April 2004 e-zine on our website. You can access our past e-zines by clicking the link below.

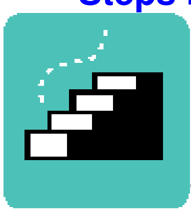
When to Use

Individuals control charts should be used when there is only one data point to represent a situation at a given time. The individuals control chart allows you to plot a point on the chart for each sample taken. This permits you to determine if the process is in statistical control or not for each sample taken. This may seem to imply that you should always use individuals charts. That is not necessarily true as is discussed below.

To use the individuals control chart, the individual sample results should be "somewhat" normally distributed. If not, the individuals control chart will give more false signals for the tests such as the zone tests, i.e., will indicate more often that the process is out of control when it actually is not.

Individuals control charts are not as sensitive to changes as Xbar-R charts. In addition, values of X and R can have significant variation (even though the process is in control) until the number of individual data points reaches 100.

Steps in Constructing



The steps in constructing the individuals control chart are given below.

1. Gather the data.

a. Select the frequency with which the data will be collected. Data should be collected in the order in which they are generated.

b.Select the number of data points (k) to be collected before control limits are calculated (at least twenty). These will be tentative limits. Control limits should be recalculated after you have collected 100 data points.

c. Record the individual sample results.

d. Calculate the moving range (R_{i+1}) between consecutive sample results:

$$R_{i+1} = |X_{i+1} - X_i|$$

where X_{i+1} is the result for sample i+1 and X_i is the result for sample i. The range value is always positive.

2. Plot the data.

a. Select the scales for the x and y axes for both the X and R charts.

b. Plot the ranges on the R chart and connect consecutive points with a straight line.

c. Plot the individual sample results on the X chart and connect consecutive points with a straight line.

3. Calculate the overall process averages and the control limits.

a. Calculate the average moving range (Rbar):

$$\overline{R} = \frac{\sum R}{k-1} = \frac{\text{Add up the R values}}{k-1}$$

Note there is always one less range value than individual sample results for the individuals control chart.

b. Plot Rbar on the range chart as a solid line and label.

c. Calculate the overall process average (Xbar):

$$\overline{X} = \frac{\sum X}{k} = \frac{\text{Add up the X values}}{k}$$

d. Plot Xbar on the X chart as a solid line and label.

e. Calculate the control limits for the R chart. The upper control limit is given by UCLr. The lower control limit is given by LCLr.

$$UCLr = 3.27\overline{R}$$

$$LCLr = \text{None}$$

f. Plot the upper control limit on the R chart as a dashed line and label.

g. Calculate the control limits for the X chart. The upper control limit is given by UCLx. The lower control limit is given by LCLx.

$$UCLx = \overline{X} + 2.66\overline{R}$$

$$LCLx = \overline{X} - 2.66\overline{R}$$

h. Plot the control limits on the X chart as dashed lines and label.

4. Interpret both charts for statistical control.

a. Always consider variation first. If the R chart is out of control, the control limits on the X chart may not be valid since you do not have a good estimate of Rbar.

b. All tests for statistical control apply to the X chart. However, the data on the range chart are not independent. Each data point is used twice. The only test that is valid for the range chart is points beyond the control limits.

5. Calculate the process standard deviation, if appropriate.

a. If the R chart is in statistical control, the process standard deviation, s, can be calculated as:

$$\sigma' = \frac{\overline{R}}{1.128}$$

Example: Waiting in Line

Waiting in line at a bank can be very frustrating. Your bank has just implemented a promise that your wait in line will never be longer than five minutes. You decide to find out if this is really true. Since you only go to the bank about once a week, you have infrequent data. You feel that the individual measurements (time waiting in line) are probably a normal distribution. You decide to use an individuals/moving range chart to determine if the bank is keeping its promise. Each time you go to the bank, you measure how long you wait in line to the nearest 0.1 minute. The results for 24 trips to the bank are given below.

3.5
2.4
4.1
2.8
3
4.7
1.2
0.9
2.5
3.1
3.6
4.1
3.8
2.5
2.8
4.3
4.1
3.6
2.4
4.8
3.5
2.5
1.3
4.5

The first step after collecting the data is to calculate the moving range between trips to the bank. For example, the moving range is the range between trips 1 and 2 is:

$$R_{i+1} = |X_{i+1} - X_i| = 3.5 - 2.4 = 1.1$$

This is done for each consecutive trips. The moving range between trips 2 and 3 is |2.4 - 4.1| = 1.7. Remember, the range is always positive.

The next step is to calculate the overall process average and the average range. The overall process average is determined by adding up the individual results for each trip and dividing by the number of samples (trips). In this case, the number of trips (k) is 24. The average range is determined by adding up the moving range values and dividing by k- 1 since there is one less range value than individual samples.

$$Xbar = 76/24 = 3.17$$

$$Rbar = 27.4/23 = 1.19$$

The next step is to calculate the control limits. The control limits are:

$$UCL_r = 3.27\ Rbar = 3.27(1.19) = 3.89$$

$$LCL_r = \text{None}$$

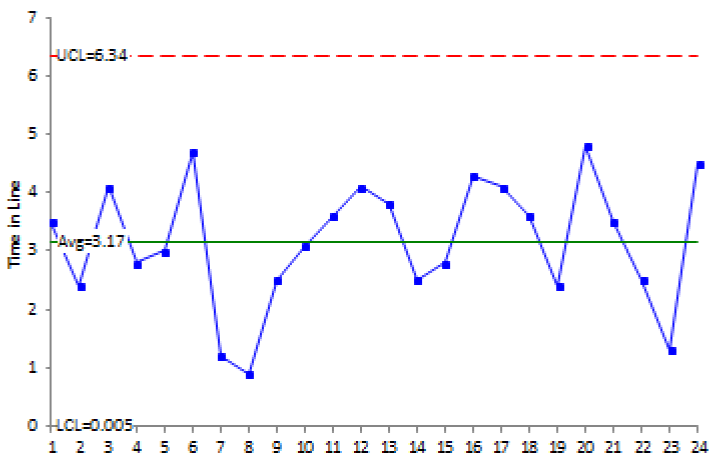
$$UCL_x = Xbar + 2.66Rbar = 3.17 + 2.66(1.19) = 6.34$$

$$LCL_x = Xbar - 2.66Rbar = 3.17 - 2.66(1.19) = 0.005$$

If the moving range chart is in control, the standard deviation of the individual results can be determined. The moving range chart (as shown below) is in control. The standard deviation is then given by:

$\sigma' = Rbar/1.128 = 1.19/1.128 = 1.05$

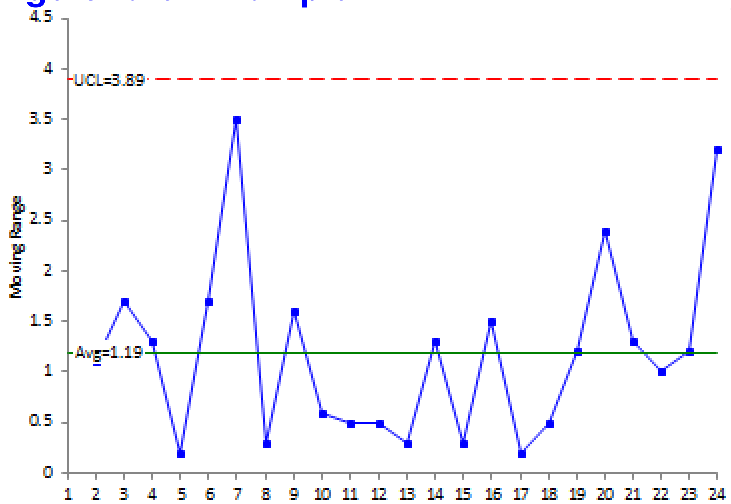
X Chart - Example



The X chart for waiting in line is shown in this example. The moving range chart is shown below. The process is in control. This means, that as long as the process stays the same, you can predict, within a range, how long you will wait in line when you go to the bank.

This means that, when you go to the bank, you will wait in line anywhere from 0 to 6.34 minutes. What does this mean about your bank's commitment that you will not wait more than 5 minutes in line? It means that the bank is not capable of meeting that guarantee. You have not waited more than 5 minutes yet based on the data. But it is just a matter of time before you do.

Moving Range Chart - Example



This e-zine has introduced individuals control charts. This type of chart should be used when data are infrequently available. The individual measurements should be normally distributed to use an individuals chart. The X chart is examining the long-term variation in individual sample results. The range chart is examining the short-term variation between consecutive sample results. Like all control charts, the individuals chart is used to determine if the process is in statistical control.