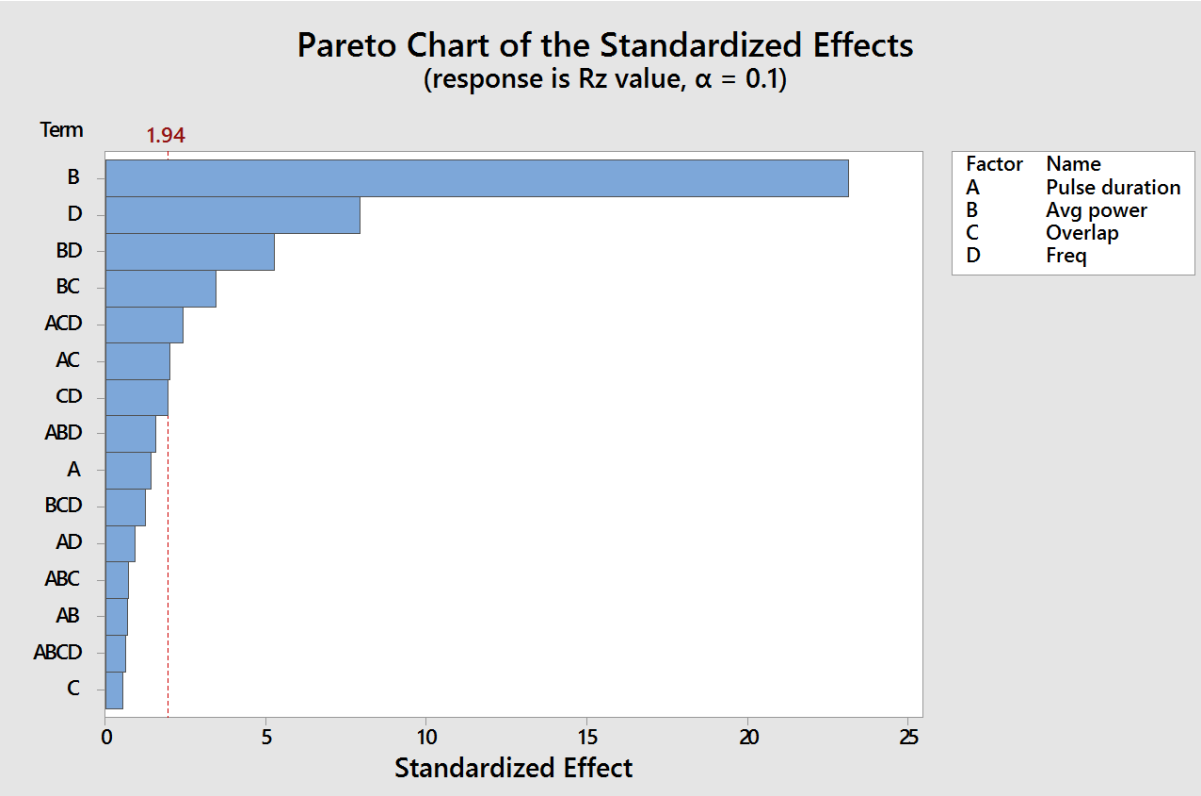
DOE ANALYSIS FOR THE Rz value

We take all the terms and perform the factor analysis. These are the significant factors as yielded by ANOVA.

STEP 1: ANOVA WITH ALL FACTORS

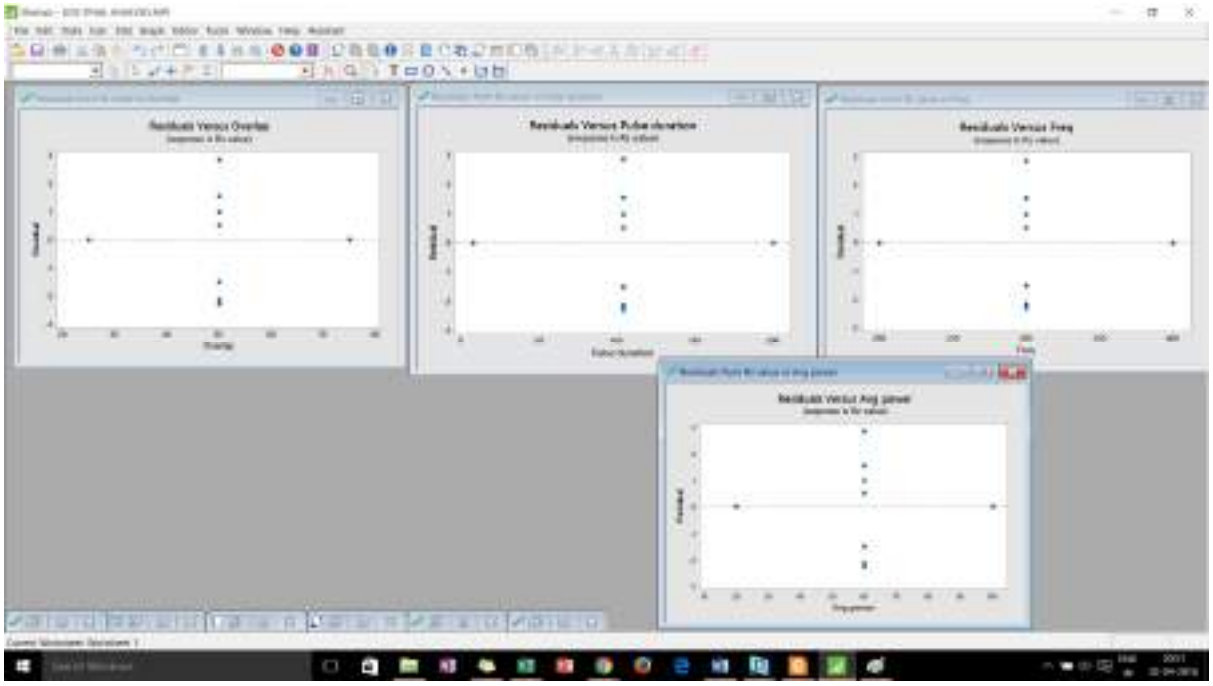


Let us see the design points which was used

C1	C2	C3	C4	C5	C6	C7	C8	C9	C10
StdOrder	RunOrder	PtType	Blocks	Pulse duration	Avg power	Overlap	Freq	Ablation depth	Rz value
31	2	0	1	104	60	50	300	5.139	6.7325
30	3	0	1	104	60	50	300	6.543	6.5886
8	4	1	1	200	100	75	200	172.181	34.7602
2	5	1	1	200	20	25	200	1.058	2.9709
1	6	1	1	8	20	25	200	0.305	2.8620
13	7	1	1	8	20	75	400	1.630	2.8769
29	8	0	1	104	60	50	300	8.506	7.3554
11	9	1	1	8	100	25	400	0.838	21.8105
3	10	1	1	8	100	25	200	16.388	37.9083
14	11	1	1	200	20	75	400	3.090	3.4436
27	12	0	1	104	60	50	300	10.642	9.8387
12	13	1	1	200	100	25	400	3.497	24.9796
4	14	1	1	200	100	25	200	29.928	33.7215
28	15	0	1	104	60	50	300	14.307	9.3729
25	16	0	1	104	60	50	300	15.569	11.7102
6	18	1	1	200	20	75	200	12.818	13.4966
5	20	1	1	8	20	75	200	9.578	4.4498
15	22	1	1	8	100	75	400	50.949	18.9417
26	25	0	1	104	60	50	300	16.216	10.4101
10	28	1	1	200	20	25	400	0.679	2.7754
7	29	1	1	8	100	75	200	147.061	30.3521
16	30	1	1	200	100	75	400	57.720	18.5460
9	31	1	1	8	20	25	400	0.883	4.0989

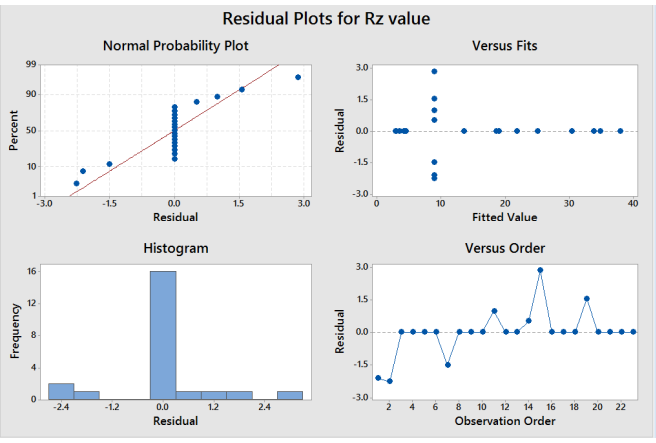
The design points were taken from the RSM conducted.

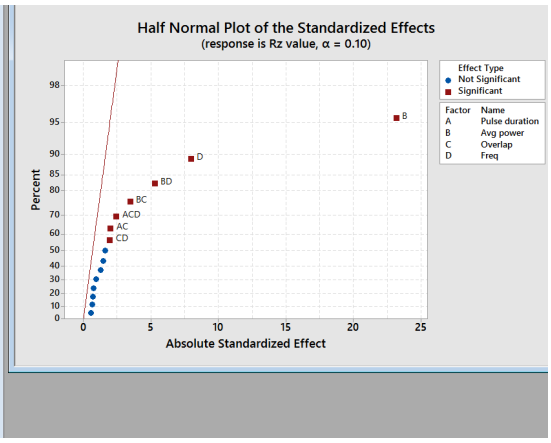
Now let us see the residual plots vs variable



As we can see from earlier there is a problem with the Rz plot since it is not scattered across the mean line. However this is not a concern as we are most interested in gaining the significant factors and we will remodel it.

Let us look at the 4 residuals graph now also





As noted not the best fit for the model. However we will check the ANOVA also.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	16	2866.79	179.17	45.37	0.000
Linear	4	2374.75	593.69	150.32	0.000
Pulse duration	1	8.11	8.11	2.05	0.202
Avg power	1	2117.06	2117.06	536.04	0.000
Overlap	1	1.13	1.13	0.29	0.611
Freq	1	248.45	248.45	62.91	0.000
2-Way Interactions	6	192.33	32.05	8.12	0.011
Pulse duration*Avg power	1	1.83	1.83	0.46	0.522
Pulse duration*Overlap	1	15.72	15.72	3.98	0.093
Pulse duration*Freq	1	3.39	3.39	0.86	0.390
Avg power*Overlap	1	46.85	46.85	11.86	0.014
Avg power*Freq	1	109.62	109.62	27.76	0.002
Overlap*Freq	1	14.92	14.92	3.78	0.100
3-Way Interactions	4	40.97	10.24	2.59	0.143
Pulse duration*Avg power*Overlap	1	2.10	2.10	0.53	0.493
Pulse duration*Avg power*Freq	1	9.71	9.71	2.46	0.168
Pulse duration*Overlap*Freq	1	23.06	23.06	5.84	0.052
Avg power*Overlap*Freq	1	6.10	6.10	1.55	0.260
4-Way Interactions	1	1.63	1.63	0.41	0.544
Pulse duration*Avg power*Overlap*Freq	1	1.63	1.63	0.41	0.544
Curvature	1	257.11	257.11	65.10	0.000
Error	6	23.70	3.95		
Total	22	2890.49			

Model Summary

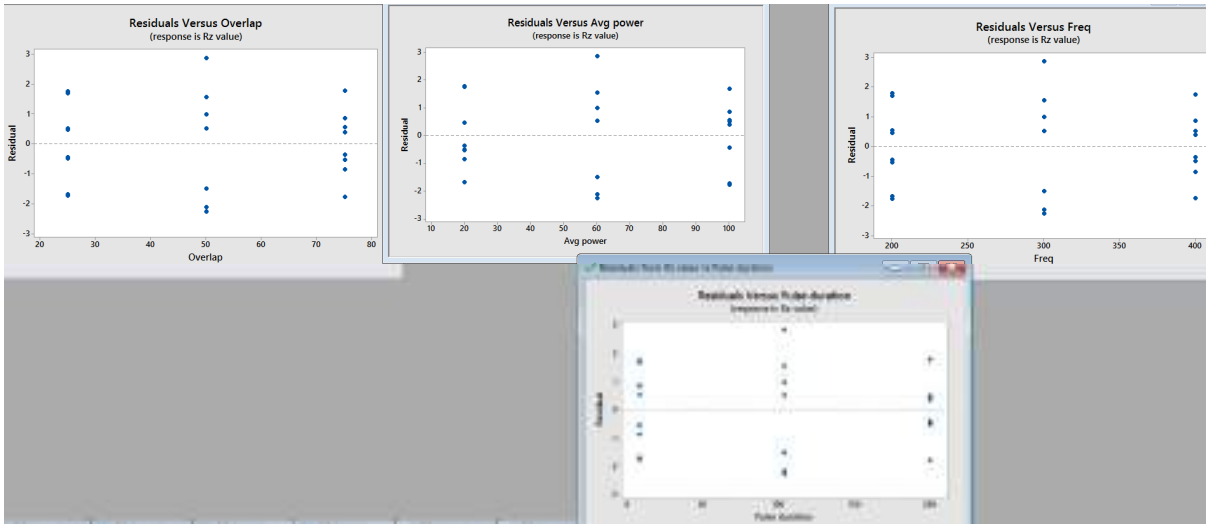
S	R-sq	R-sq(adj)	R-sq(pred)
1.98732	99.18%	96.99%	*

We get the information there is strong curvature. Rsq cannot be predicted because the degree of freedom are full. So let us now remodel the terms.

While remodelling due to hierarchy we have to include terms A, C and AD extra even though they are not significant to maintain the hierarchy.

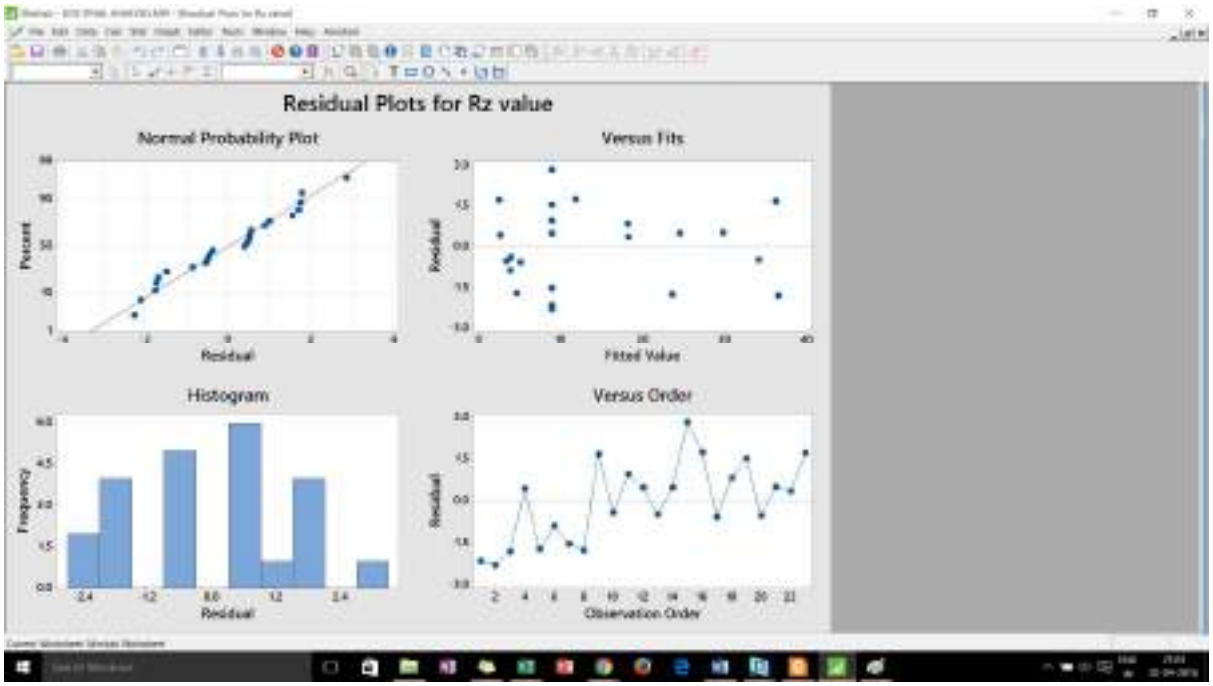
STEP 2: IMPROVING MODEL BY USING TERMS P<0.1

Residuals vs Variable plot



The residuals against variable show a random order which looks ok and meets equal variance condition.

4 graphs residuals



The histogram plot look a little normal. The residual vs fitted value looks more or less random. Do not worry about residual vs order as they have been taken from the RSM data points. We will look at them more closely during the RSM analysis.

ANOVA analysis

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	11	2845.42	258.67	63.13	0.000
Linear	4	2374.75	593.69	144.90	0.000
Pulse duration	1	8.11	8.11	1.98	0.187
Avg power	1	2117.06	2117.06	516.69	0.000
Overlap	1	1.13	1.13	0.28	0.609
Freq	1	248.45	248.45	60.64	0.000
2-Way Interactions	5	190.50	38.10	9.30	0.001
Pulse duration*Overlap	1	15.72	15.72	3.84	0.076
Pulse duration*Freq	1	3.39	3.39	0.83	0.383
Avg power*Overlap	1	46.85	46.85	11.43	0.006
Avg power*Freq	1	109.62	109.62	26.75	0.000
Overlap*Freq	1	14.92	14.92	3.64	0.083
3-Way Interactions	1	23.06	23.06	5.63	0.037
Pulse duration*Overlap*Freq	1	23.06	23.06	5.63	0.037
Curvature	1	257.11	257.11	62.75	0.000
Error	11	45.07	4.10		
Lack-of-Fit	5	21.37	4.27	1.08	0.454
Pure Error	6	23.70	3.95		
Total	22	2890.49			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
2.02419	98.44%	96.88%	91.31%

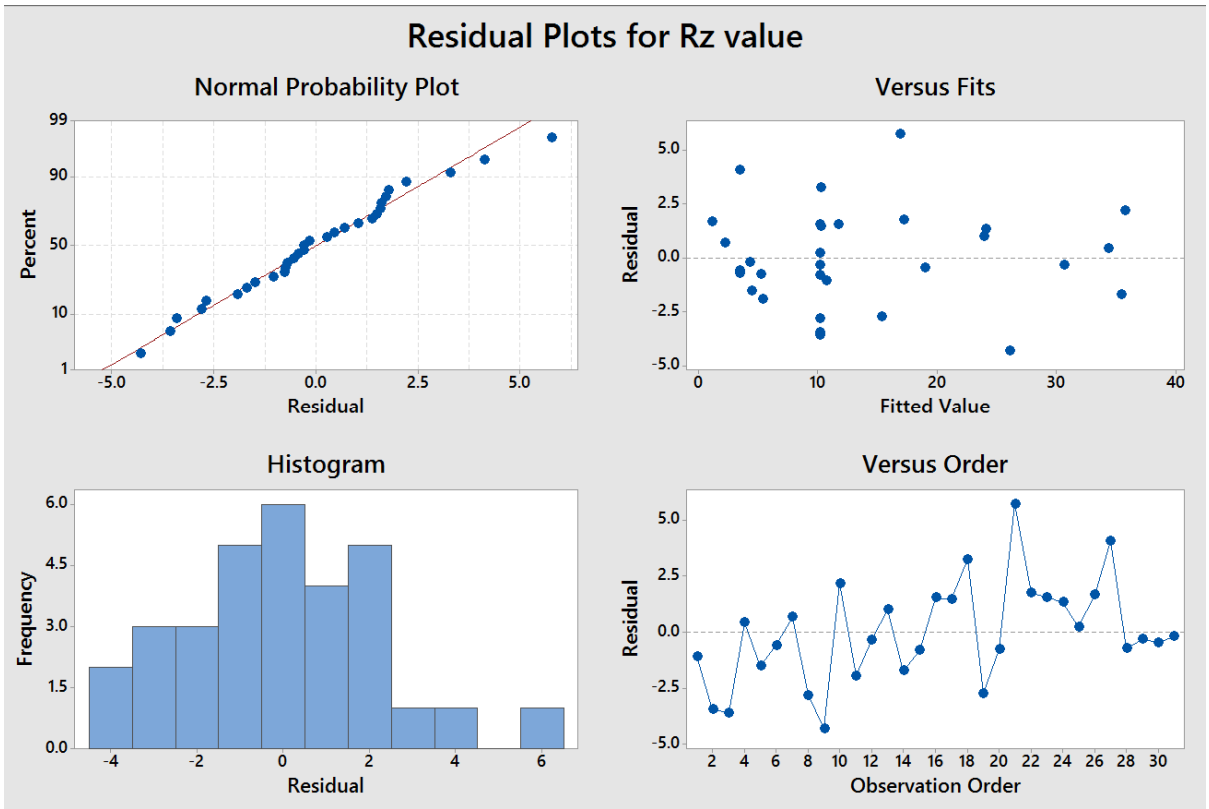
The lack of fit shows that the model fits the data quite ok. Maybe it could be improved, however since there is curvature we will go ahead and conduct the RSM and try to figure out that square term. We also know the significant terms which will help in improving the RSM model.

But it is good to note the linear model generated by the ANOVA analysis

```
Rz value = -3.80 - 0.0739 Pulse duration + 0.5694 Avg power + 0.0089 Overlap - 0.0019 Freq
           + 0.001914 Pulse duration*Overlap + 0.000202 Pulse duration*Freq
           - 0.001711 Avg power*Overlap - 0.000654 Avg power*Freq + 0.000134 Overlap*Freq
           - 0.000005 Pulse duration*Overlap*Freq - 7.266 Ct Pt
```

RSM of Rz Analysis

Step 3: RSM OF Rz FUNCTION WITH QUADRATIC FUNCTION WITH ALL TERMS



The residual 4 graphs show that the histogram is fairly normal shaped. The Residual vs fits looks random and the residual vs observations looks ok. Let us check the ANOVA analysis also.

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	3135.53	223.97	23.50	0.000
Linear	4	2629.47	657.37	68.97	0.000
Pulse duration	1	9.30	9.30	0.98	0.338
Avg power	1	2373.17	2373.17	248.97	0.000
Overlap	1	11.37	11.37	1.19	0.291
Freq	1	235.63	235.63	24.72	0.000
Square	4	313.73	78.43	8.23	0.001
Pulse duration*Pulse duration	1	1.83	1.83	0.19	0.667
Avg power*Avg power	1	15.51	15.51	1.63	0.220
Overlap*Overlap	1	92.30	92.30	9.68	0.007
Freq*Freq	1	24.85	24.85	2.61	0.126
2-Way Interaction	6	192.33	32.05	3.36	0.024
Pulse duration*Avg power	1	1.83	1.83	0.19	0.668
Pulse duration*Overlap	1	15.72	15.72	1.65	0.217
Pulse duration*Freq	1	3.39	3.39	0.36	0.560
Avg power*Overlap	1	46.85	46.85	4.92	0.041
Avg power*Freq	1	109.62	109.62	11.50	0.004
Overlap*Freq	1	14.92	14.92	1.57	0.229
Error	16	152.51	9.53		
Lack-of-Fit	10	128.81	12.88	3.26	0.081
Pure Error	6	23.70	3.95		
Total	30	3288.04			

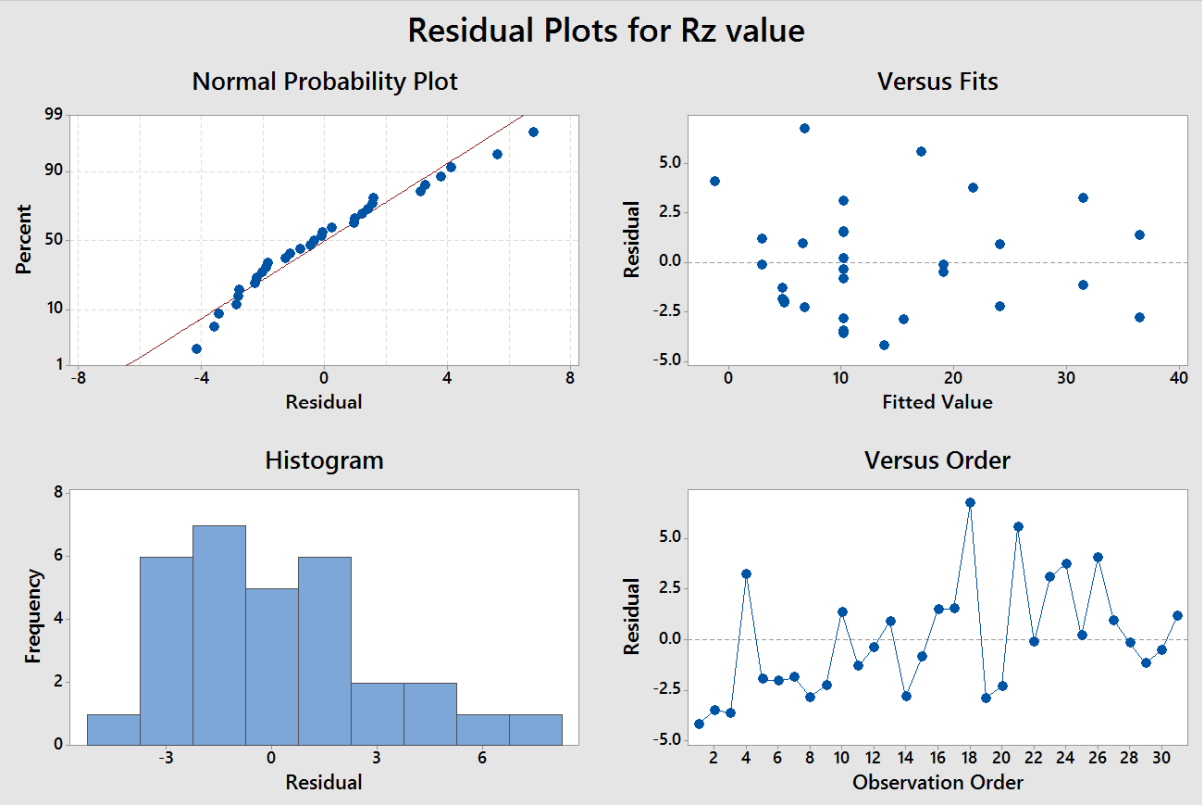
Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
3.08738	95.36%	91.30%	78.20%

The anova shows that the model fits in ok. Let see if we can improve the model further by removing the non significant interaction terms. We remove all pulse duration terms and check for the residuals and prediction model.

[THE RESIDUALS SEEMS TO FIT WELL, IS IT NECESSARY TO FURTHER IMPROVE THIS MODEL??]

STEP 4: TRYING TO IMPROVE MODEL BY REMOVING TERMS WITH P>0.1



The Histogram graph is showing slight skewdness which can maybe removed through a certain transformation. Residual vs order is showing a slight trend but I think is ok. Let us also check the ANOVA now.

Backward Elimination of Terms

α to remove = 0.1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	6	3057.81	509.63	53.13	0.000
Linear	3	2620.17	873.39	91.05	0.000
Avg power	1	2373.17	2373.17	247.39	0.000
Overlap	1	11.37	11.37	1.19	0.287
Freq	1	235.63	235.63	24.56	0.000
Square	1	281.16	281.16	29.31	0.000
Overlap*Overlap	1	281.16	281.16	29.31	0.000
2-Way Interaction	2	156.47	78.24	8.16	0.002
Avg power*Overlap	1	46.85	46.85	4.88	0.037
Avg power*Freq	1	109.62	109.62	11.43	0.002
Error	24	230.23	9.59		
Lack-of-Fit	18	206.53	11.47	2.91	0.096
Pure Error	6	23.70	3.95		
Total	30	3288.04			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
3.09723	93.00%	91.25%	88.11%

What we see is that the model fits in a little bit better than before R pred has slightly increased.

Response Surface Regression: Rz value versus Avg power, Overlap, Freq

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	6	3057.81	509.63	53.13	0.000
Linear	3	2620.17	873.39	91.05	0.000
Avg power	1	2373.17	2373.17	247.39	0.000
Overlap	1	11.37	11.37	1.19	0.287
Freq	1	235.63	235.63	24.56	0.000
Square	1	281.16	281.16	29.31	0.000
Overlap*Overlap	1	281.16	281.16	29.31	0.000
2-Way Interaction	2	156.47	78.24	8.16	0.002
Avg power*Overlap	1	46.85	46.85	4.88	0.037
Avg power*Freq	1	109.62	109.62	11.43	0.002
Error	24	230.23	9.59		
Lack-of-Fit	18	206.53	11.47	2.91	0.096
Pure Error	6	23.70	3.95		
Total	30	3288.04			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
3.09723	93.00%	91.25%	88.11%

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	10.190	0.859	11.86	0.000	
Avg power	11.482	0.730	15.73	0.000	1.00
Overlap	-0.795	0.730	-1.09	0.287	1.00
Freq	-3.618	0.730	-4.96	0.000	1.00
Overlap*Overlap	6.10	1.13	5.41	0.000	1.00
Avg power*Overlap	-1.711	0.774	-2.21	0.037	1.00
Avg power*Freq	-2.617	0.774	-3.38	0.002	1.00

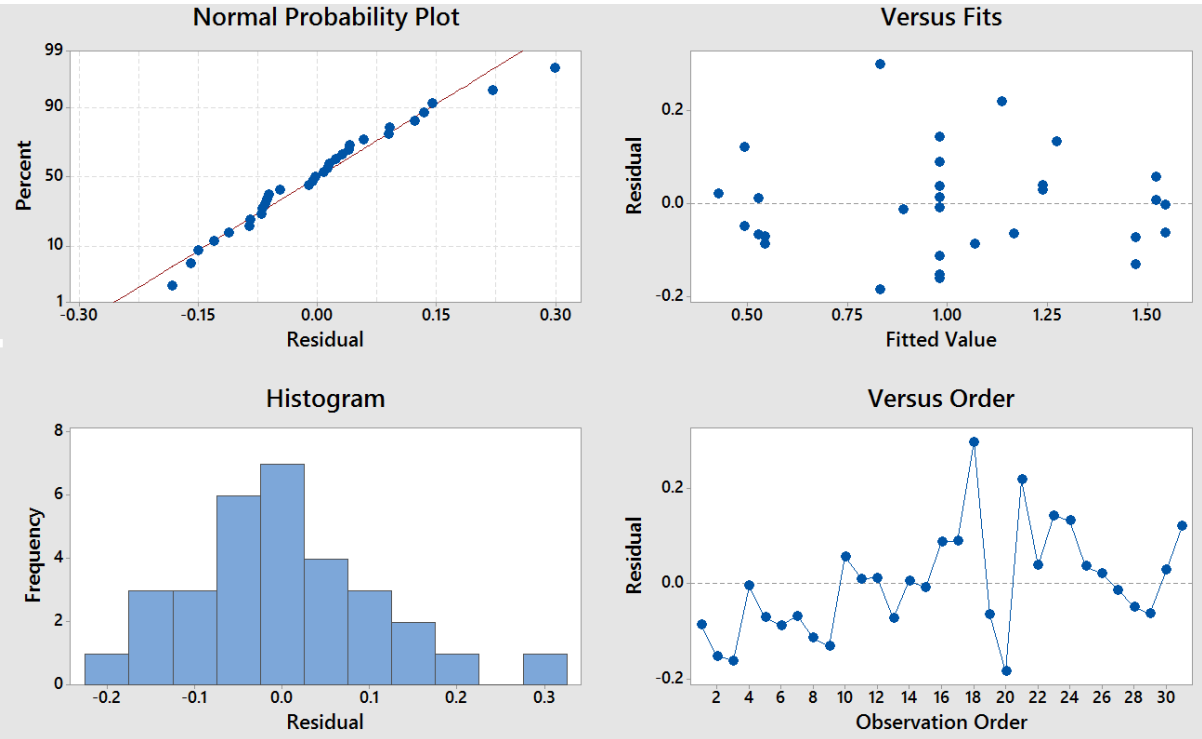
We see the R(prediction) has increased slightly with not much loss in Rsq adjusted. However the next step could be to transform the response function for removing skewdness in the histogram.

A log transformation looks like a possible step that can be explored. Here is the Log10 transform of the response Rz and the design table shown once again. However instead of first analysing with all variables and then reducing it subsequently to improve it, there is an option called backward elimination in MINITAB which does the same function avoiding the iterations required. Here are the results after backward elimination.

STEP 5: DIRECT LOG TRANSFORMATION OF THE RESPONSE FUNCTION AND AGAIN TRYING TO FIT QUADRATIC TERMS, BACKWARD ELIMINATION OF P>0.1 INCLUDED

StdOrder	RunOrder	PtType	Blocks	Pulse duration	Avg power	Overlap	Freq	Ablation rate	Rz value	log Rz
23	1	-1	1	104	60	50	200	21.261	9.6255	0.98342
31	2	0	1	104	60	50	300	5.139	6.7325	0.82818
30	3	0	1	104	60	50	300	6.543	6.5886	0.81879
8	4	1	1	200	100	75	200	172.181	34.7602	1.54108
2	5	1	1	200	20	25	200	1.058	2.9709	0.47288
1	6	1	1	8	20	25	200	0.305	2.8620	0.45666
13	7	1	1	8	20	75	400	1.630	2.8769	0.45892
29	8	0	1	104	60	50	300	8.506	7.3554	0.86661
11	9	1	1	8	100	25	400	0.838	21.8105	1.33867
3	10	1	1	8	100	25	200	16.388	37.9083	1.57873
14	11	1	1	200	20	75	400	3.090	3.4436	0.53702
27	12	0	1	104	60	50	300	10.642	9.8387	0.99294
12	13	1	1	200	100	25	400	3.497	24.9796	1.39758
4	14	1	1	200	100	25	200	29.928	33.7215	1.52791
28	15	0	1	104	60	50	300	14.307	9.3729	0.97187
25	16	0	1	104	60	50	300	15.569	11.7102	1.06856
17	17	-1	1	8	60	50	300	13.393	11.7490	1.07000
6	18	1	1	200	20	75	200	12.818	13.4966	1.13022
22	19	-1	1	104	60	75	300	55.127	12.6226	1.10115
5	20	1	1	8	20	75	200	9.578	4.4498	0.64834
21	21	-1	1	104	60	25	300	8.682	22.6681	1.35542
15	22	1	1	8	100	75	400	50.949	18.9417	1.27742
18	23	-1	1	200	60	50	300	18.955	13.2934	1.12364
20	24	-1	1	104	100	50	300	38.129	25.4433	1.40557
26	25	0	1	104	60	50	300	16.216	10.4101	1.01745
19	26	-1	1	104	20	50	300	1.486	2.8083	0.44844
24	27	-1	1	104	60	50	400	6.823	7.5482	0.87784
10	28	1	1	200	20	25	400	0.679	2.7754	0.44332
7	29	1	1	8	100	75	200	147.061	30.3521	1.48219
16	30	1	1	200	100	75	400	57.720	18.5460	1.26825
9	31	1	1	8	20	25	400	0.883	4.0989	0.61266

The 4 residual graph after backward elimination



Let us look at the ANOVA table

Backward Elimination of Terms

α to remove = 0.1

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	7	3.60108	0.51444	32.28	0.000
Linear	3	3.36418	1.12139	70.37	0.000
Avg power	1	3.21644	3.21644	201.83	0.000
Overlap	1	0.00378	0.00378	0.24	0.631
Freq	1	0.14396	0.14396	9.03	0.006
Square	2	0.10174	0.05087	3.19	0.060
Avg power*Avg power	1	0.05880	0.05880	3.69	0.067
Overlap*Overlap	1	0.10159	0.10159	6.37	0.019
2-Way Interaction	2	0.13516	0.06758	4.24	0.027
Avg power*Overlap	1	0.07061	0.07061	4.43	0.046
Overlap*Freq	1	0.06455	0.06455	4.05	0.056
Error	23	0.36654	0.01594		
Lack-of-Fit	17	0.30765	0.01810	1.84	0.231
Pure Error	6	0.05889	0.00982		
Total	30	3.96762			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.126240	90.76%	87.95%	81.40%

The lack of fit P value has substantially increased showing that this model fits much better than the previous models done in RSM. So I think the model is ready to use.

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	0.9795	0.0365	26.81	0.000	
Avg power	0.4227	0.0298	14.21	0.000	1.00
Overlap	0.0145	0.0298	0.49	0.631	1.00
Freq	-0.0894	0.0298	-3.01	0.006	1.00
Avg power*Avg power	-0.1302	0.0678	-1.92	0.067	2.18
Overlap*Overlap	0.1711	0.0678	2.52	0.019	2.18
Avg power*Overlap	-0.0664	0.0316	-2.10	0.046	1.00
Overlap*Freq	-0.0635	0.0316	-2.01	0.056	1.00

Regression Equation in Uncoded Units

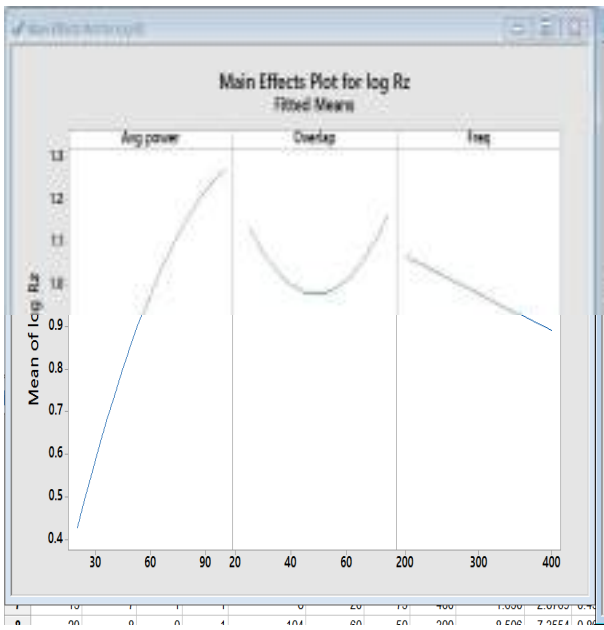
log Rz = 0.396 + 0.02365 Avg power - 0.0152 Overlap + 0.000376 Freq
- 0.000081 Avg power*Avg power + 0.000274 Overlap*Overlap
- 0.000066 Avg power*Overlap - 0.000025 Overlap*Freq

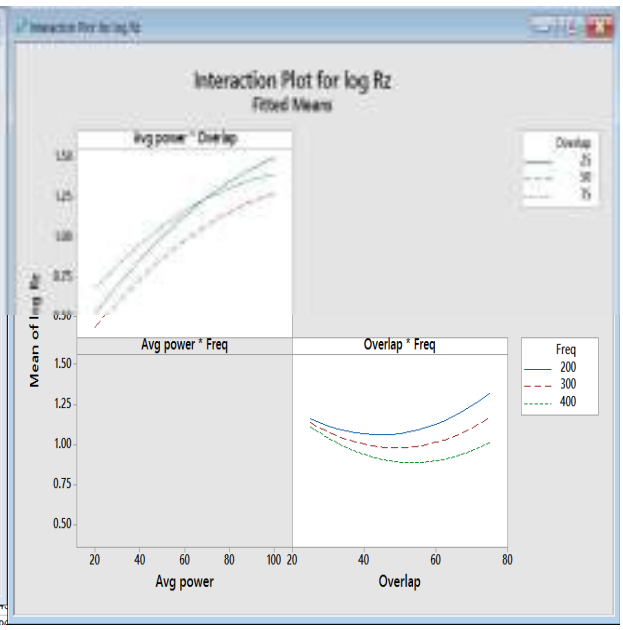
Fits and Diagnostics for Unusual Observations

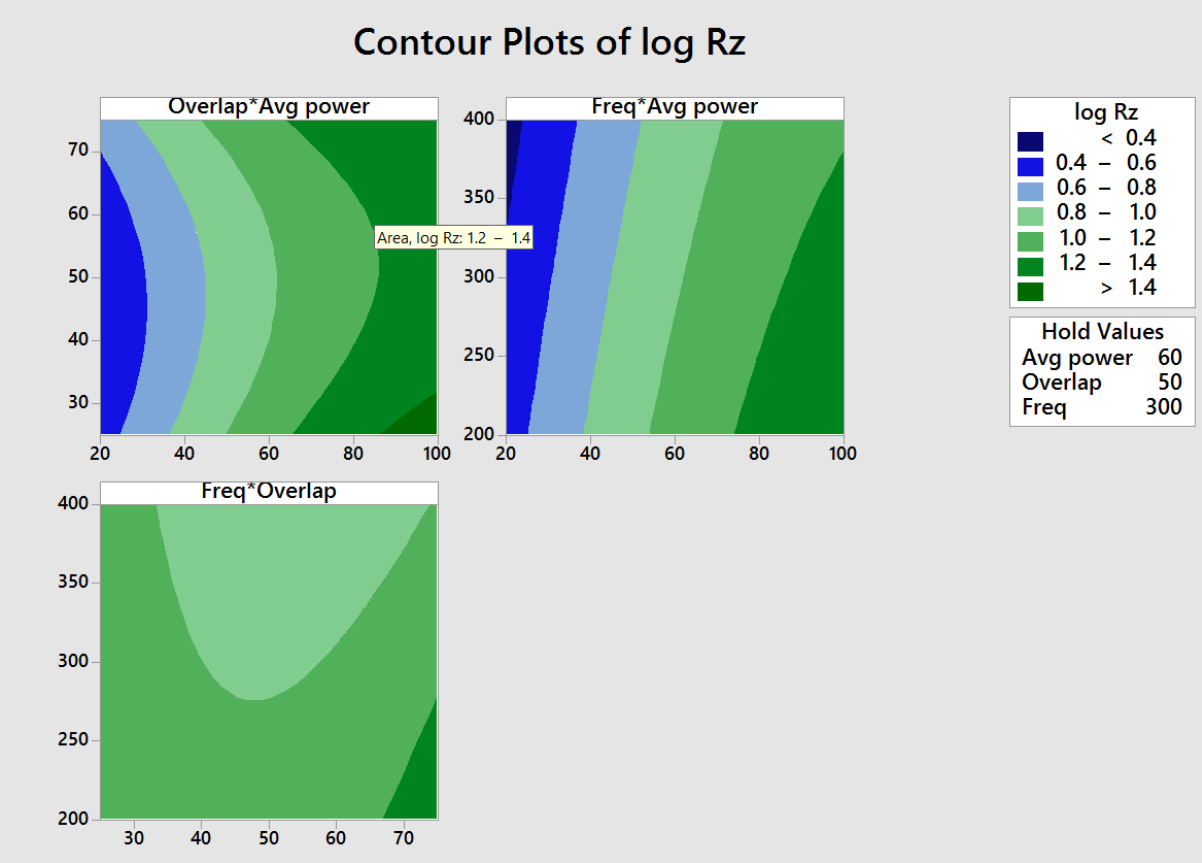
Obs	log Rz	Fit	Resid	Std Resid	
18	1.1302	0.8316	0.2986	2.94	R
21	1.3554	1.1361	0.2193	2.14	R

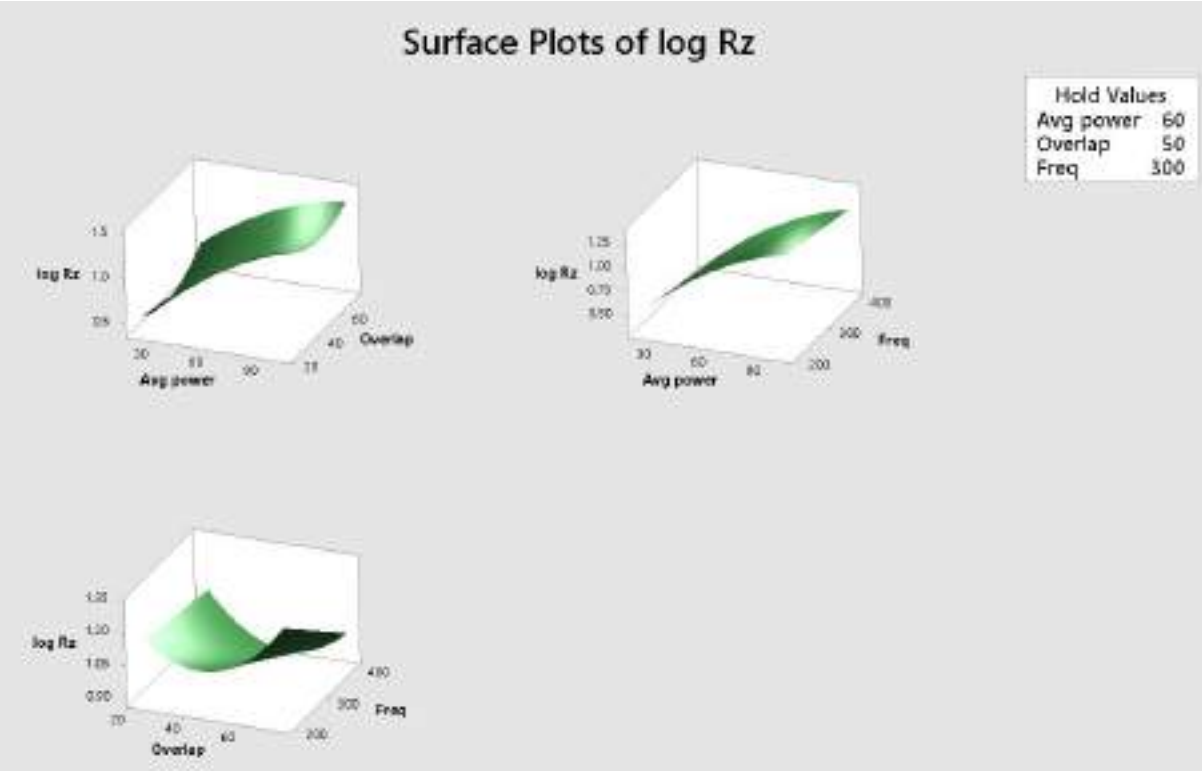
The regression equation is clearly shown. The outlier has been left as it is. It has been already remeasured thrice.

Now we can utilize this function for Response optimizer and confirming the runs.






Contour Plots of log Rz
Overlap*Avg power
Freq*Avg power
Freq*Overlap
Area, log Rz: 1.2 – 1.4
log Rz
< 0.4
0.4 – 0.6
0.6 – 0.8
0.8 – 1.0
1.0 – 1.2
1.2 – 1.4
> 1.4
Hold Values
Avg power 60
Overlap 50
Freq 300


Surface Plots of log Rz
Hold Values
Avg power 60
Overlap 50
Freq 300
log Rz
Avg power
Overlap
Freq