ANALYSIS OF THE RESULTS FOR JUMMA MACHINE

Objectives of the DOE

- Primary objective: Find the significant factors affecting the response function in this case Ablation depth
- Secondary Objective: Devise a model for the relationship between Response function and Ablation depth
- Tertiary Objective: Confirm the model with confirmatory runs

1. Steps conducted

a. DOE conducted with 4 factors to find out significant factors (16 runs)

b. Try to gauge the important factors and detect curvatures (7 center points)

c. Conducted 8 more experiments to model an equation for the response found. (8 axial points augmented on DOE to model ablation depth using face centered method)

Following are the Results for **ABLATION DEPTH** (response function)

Factors affecting the Ablation depth are

1. Pulse duration

2. Overlap

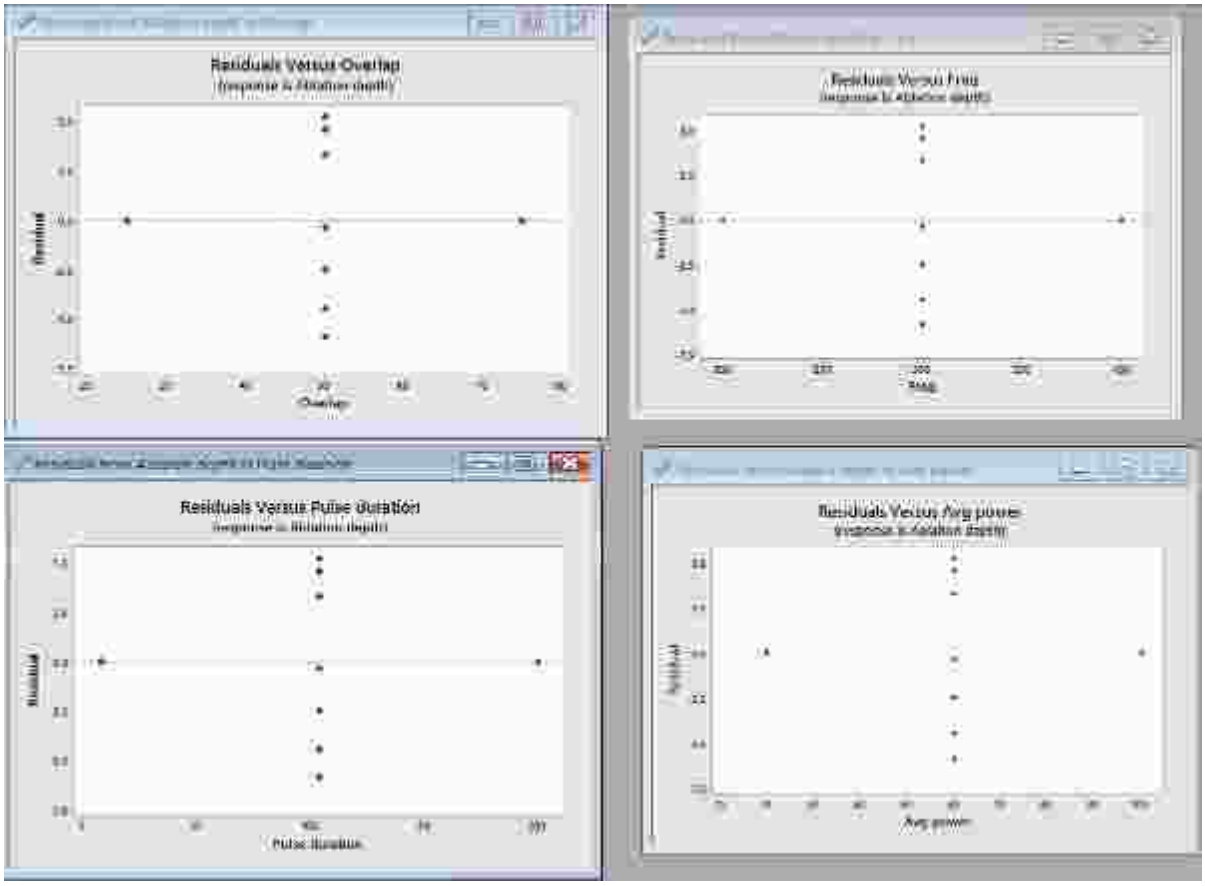
3. Freq

4. Avg Power

The factorial analysis was conducted with all terms included. Here are the results

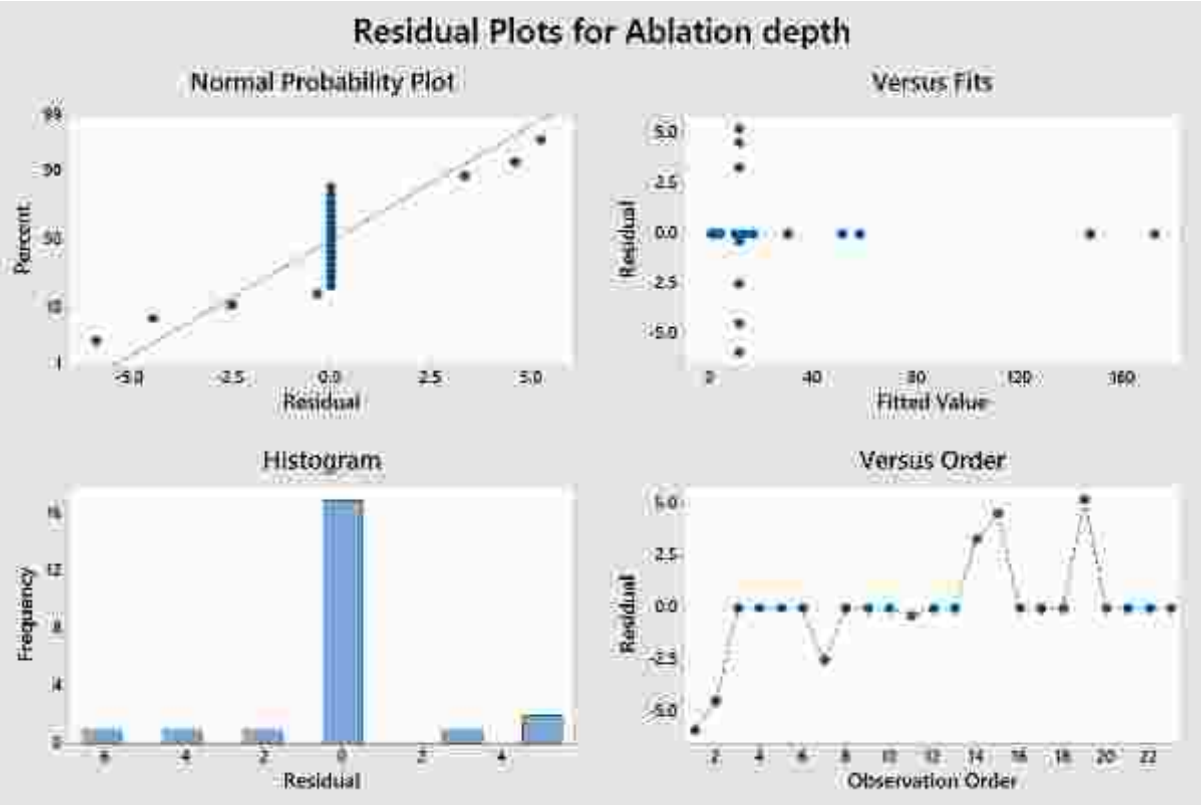
STEP 1: FACTORIAL ANALYSIS WITH ALL VARIABLES

Residual vs Variables



Conclusion – there is a problem as the residuals are not random for all levels

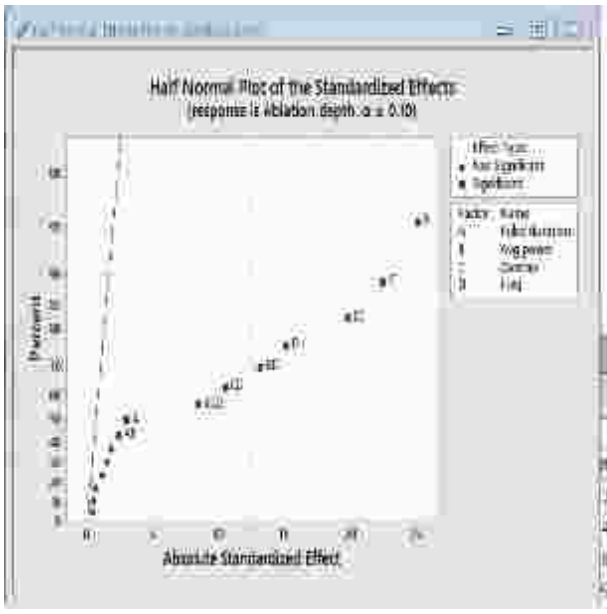
Residuals vs Fitted, Histogram, Normality of residuals and Residual vs order

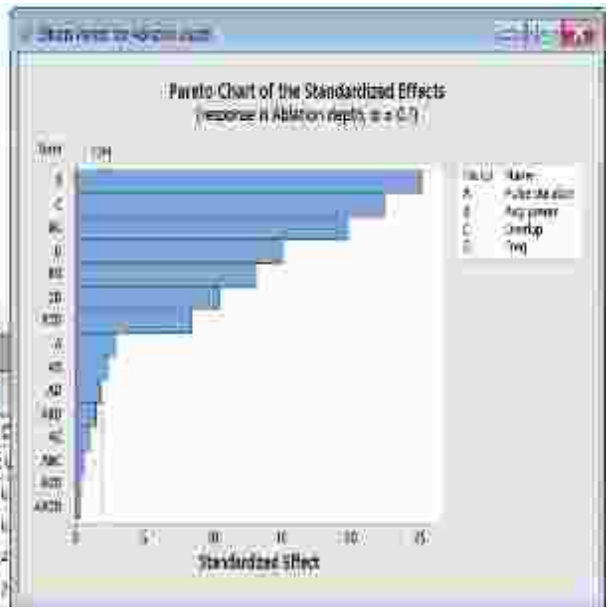


Conclusion – the model cannot be utilized and residuals vs fitted show concentration on the left.

Data points sheet

↓	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13
	StdOrder	RunOrder	PtType	Blocks	Pulse duration	Avg power	Overlap	Freq	Ablation depth	Rz value	FITS1	RESI1	
1	31	2	0	1	104	60	50	300	5.139	6.7325	10.989	-5.85007	
2	30	3	0	1	104	60	50	300	6.543	6.5886	10.989	-4.44563	
3	8	4	1	1	200	100	75	200	172.181	34.7602	172.181	-0.00000	
4	2	5	1	1	200	20	25	200	1.058	2.9709	1.058	-0.00000	
5	1	6	1	1	8	20	25	200	0.305	2.8620	0.305	-0.00000	
6	13	7	1	1	8	20	75	400	1.630	2.8769	1.630	-0.00000	
7	29	8	0	1	104	60	50	300	8.506	7.3554	10.989	-2.48290	
8	11	9	1	1	8	100	25	400	0.838	21.8105	0.838	0.00000	
9	3	10	1	1	8	100	25	200	16.388	37.9083	16.388	0.00000	
10	14	11	1	1	200	20	75	400	3.090	3.4436	3.090	-0.00000	
11	27	12	0	1	104	60	50	300	10.642	9.8387	10.989	-0.34691	
12	12	13	1	1	200	100	25	400	3.497	24.9796	3.497	-0.00000	
13	4	14	1	1	200	100	25	200	29.928	33.7215	29.928	0.00000	
14	28	15	0	1	104	60	50	300	14.307	9.3729	10.989	3.31815	
15	25	16	0	1	104	60	50	300	15.569	11.7102	10.989	4.58045	
16	6	18	1	1	200	20	75	200	12.818	13.4966	12.818	-0.00000	
17	5	20	1	1	8	20	75	200	9.578	4.4498	9.578	-0.00000	
18	15	22	1	1	8	100	75	400	50.949	18.9417	50.949	0.00000	
19	26	25	0	1	104	60	50	300	16.216	10.4101	10.989	5.22691	
20	10	28	1	1	200	20	25	400	0.679	2.7754	0.679	0.00000	
21	7	29	1	1	8	100	75	200	147.061	30.3521	147.061	0.00000	
22	16	30	1	1	200	100	75	400	57.720	18.5460	57.720	0.00000	
23	9	31	1	1	8	20	25	400	0.883	4.0989	0.883	0.00000	
24													





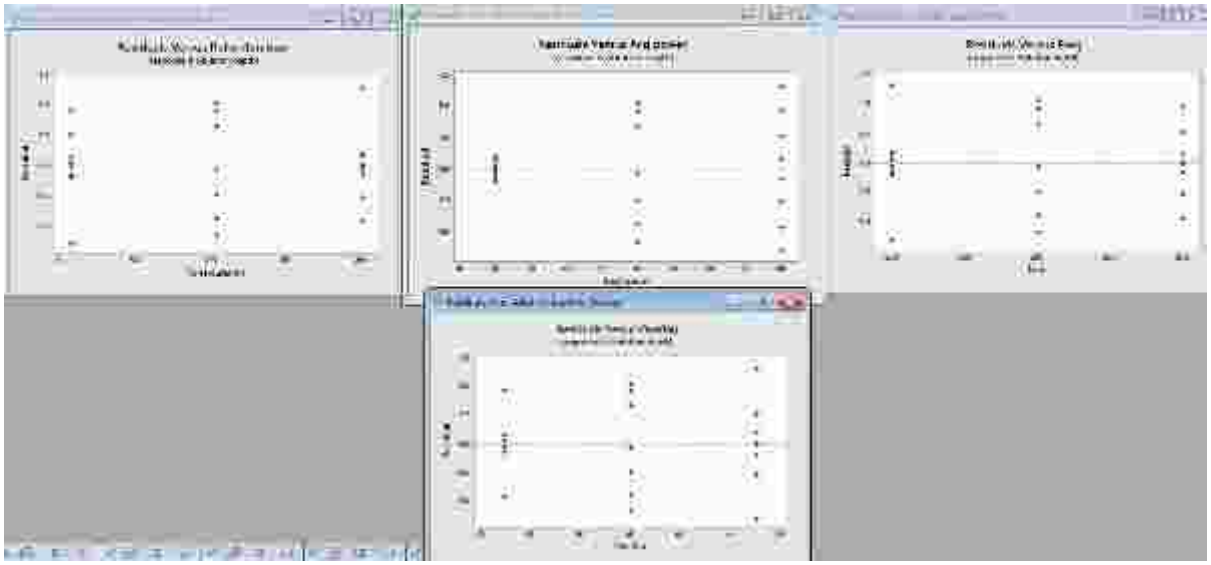
Terms which show the most significant terms.

Conclusion: Remodel with only the significant terms

STEP 2: RE-ANALYZE WITH SIGNIFICANT TERMS IE (P<0.1)

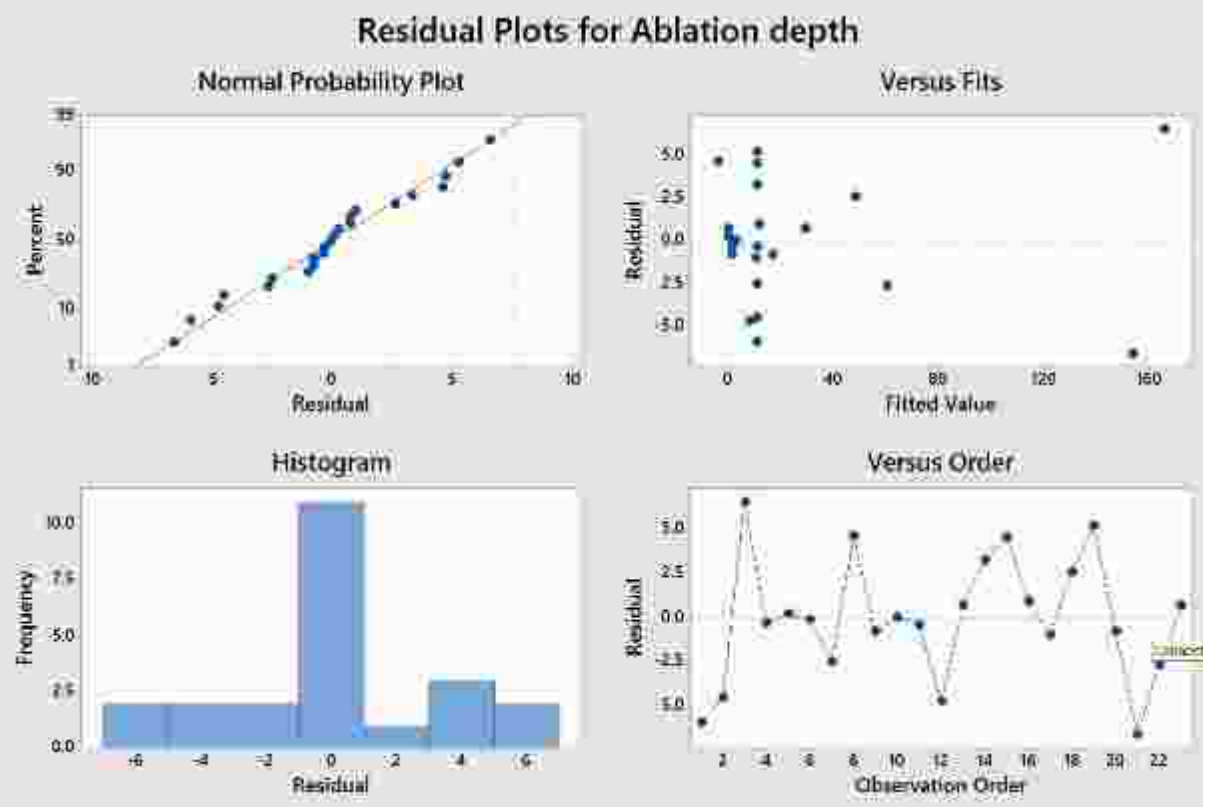
Now the model was remodeled with only the significant terms

Residuals vs variable



Conclusion- Residual vs avg power shows an increasing trend, which can be got ridden off through a transformation

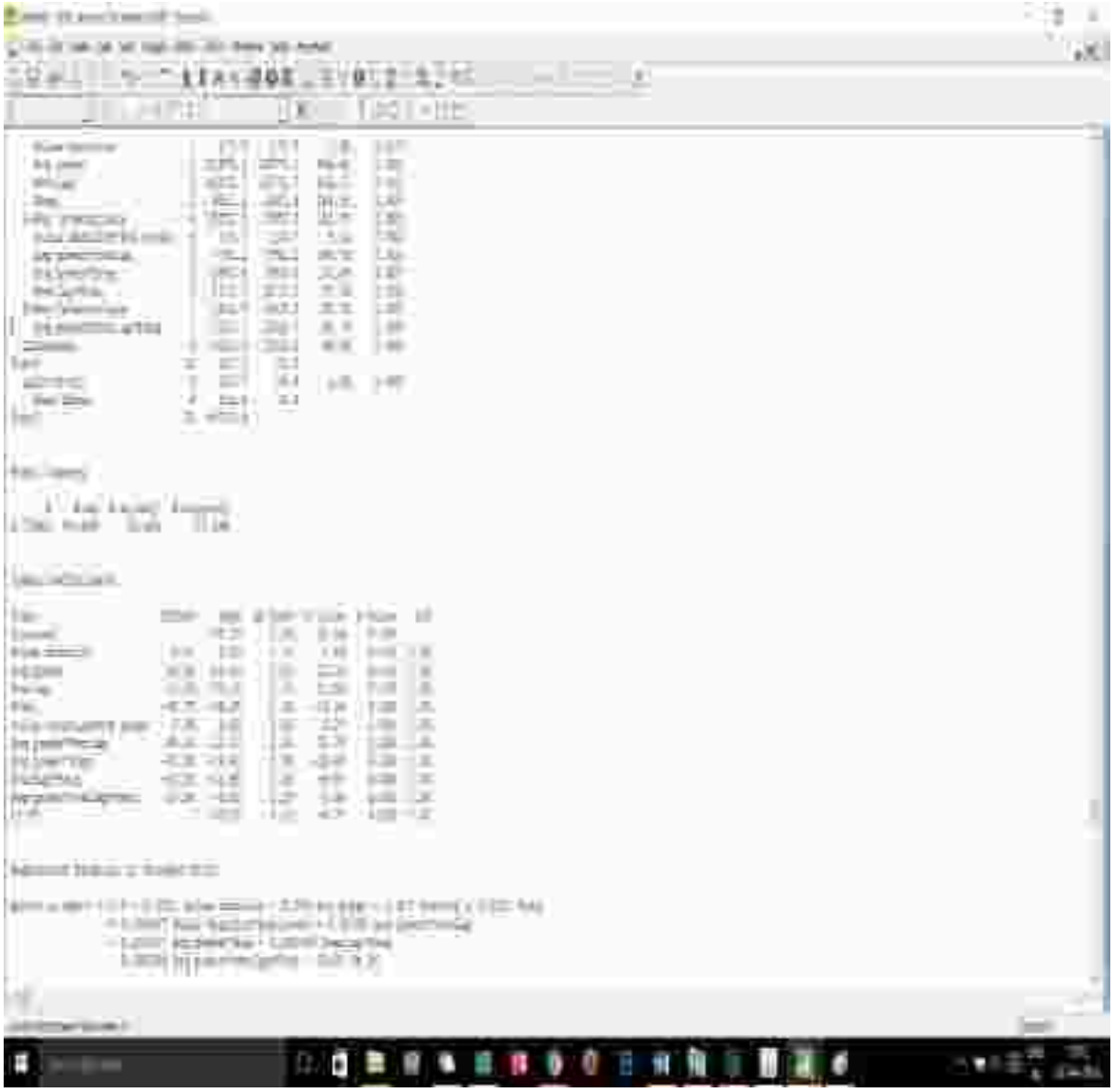
4 in 1 residuals graphs after remodelling with only significant terms



Conclusion – The histogram plot seems acceptable, residual vs order is random which is indicating no time trend.

However the residual vs fit shows a clustering for the low values and 2 outliers at the right side. The outliers were not deleted.

ANOVA checking



Model seems to fit well but cannot predict for increasing values after ablation depth of 70 since it is not well distributed. **However what we can conclude are the significant factors from the DOE have been found for the ablation depth – please see coded coefficients above.**

(Would this statement be correct since significant factors are calculated from the t-test?? Since residual analysis would be more for model fitting, so it doesn't matter about the residual graphs at all for finding significant terms??)

However since the residual vs fits was troubling I transformed the response function using square root. Here are the following results

STEP 2A: AN EXTRA SQR ROOT TRANSFORMATION WAS TRIED (NOT NECESSARY I THINK)

AFTER SQUARE ROOT TRANSFORMATION

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	8	244.368	30.5460	116.00	0.000
Linear	4	197.099	49.2748	187.12	0.000
Pulse duration	1	1.663	1.6630	6.32	0.025
Avg power	1	96.204	96.2038	365.34	0.000
Overlap	1	72.439	72.4394	275.09	0.000
Freq	1	26.793	26.7928	101.75	0.000
2-Way Interactions	3	43.892	14.6308	55.56	0.000
Avg power*Overlap	1	28.396	28.3956	107.83	0.000
Avg power*Freq	1	11.884	11.8836	45.13	0.000
Overlap*Freq	1	3.613	3.6132	13.72	0.002
Curvature	1	3.376	3.3762	12.82	0.003
Error	14	3.687	0.2633		
Lack-of-Fit	8	0.759	0.0948	0.19	0.981
Pure Error	6	2.928	0.4880		
Total	22	248.054			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.513153	98.51%	97.66%	97.17%

Coded Coefficients for Transformed Response

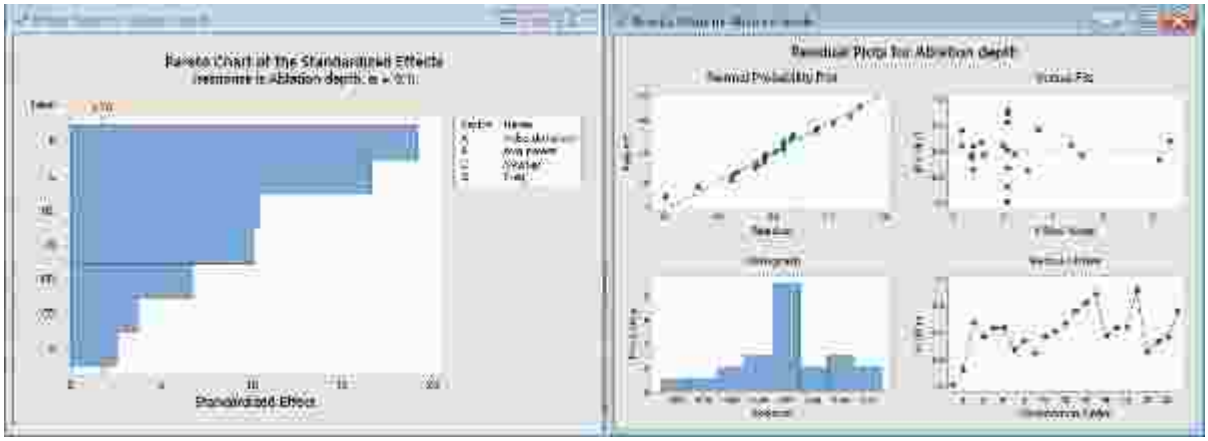
Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		4.084	0.128	31.83	0.000	
Pulse duration	0.645	0.322	0.128	2.51	0.025	1.00
Avg power	4.904	2.452	0.128	19.11	0.000	1.00
Overlap	4.256	2.128	0.128	16.59	0.000	1.00
Freq	-2.588	-1.294	0.128	-10.09	0.000	1.00
Avg power*Overlap	2.664	1.332	0.128	10.38	0.000	1.00
Avg power*Freq	-1.724	-0.862	0.128	-6.72	0.000	1.00
Overlap*Freq	-0.950	-0.475	0.128	-3.70	0.002	1.00
Ct Pt		-0.833	0.233	-3.58	0.003	1.00

Regression Equation in Uncoded Units

```
Ablation depth^0.5 = -3.05 + 0.00336 Pulse duration + 0.0593 Avg power + 0.0622 Overlap
                     + 0.00949 Freq + 0.001332 Avg power*Overlap - 0.000215 Avg power*Freq
                     - 0.000190 Overlap*Freq - 0.833 Ct Pt
```

The significant terms BCD, AB are not relevant anymore after square root transformation. Thus can I conclude that finding significant factors should be done before transforming the function ????

The 4 in 1 residual plot



Conclusion: Thus since the residual vs fitted has a more random pattern here, we can say that the model is fitting well and against the square root of the ablation depth, the following pareto charts shows the significant factors.

3 confirmatory runs not in the design points were conducted and following were the results

Run details

A. Trial 1 – Actual reading - **<u>70.1848 [200ns 60 power 75 overlap 200 freq]</u>**

Predicted values with 95% prediction interval and 95% confidence interval

```
Fit        95% CI              95% PI
68.9456  (59.1068, 79.5415)  (49.5961, 91.4749)
```

B. Trial 2 – Actual reading – **<u>6.00873 [8ns 60 power 25overlap 200 freq]</u>**

Predicted values with 95% prediction interval and 95% confidence interval

```
   Fit        95% CI              95% PI
6.01507  (3.37570, 9.41152)  (1.42006, 13.7898)
```

C. Trial 3 – Actual reading – **<u>36.75709 * [100ns 100 power 75 overlap 400 freq]</u>**

Predicted values with 95% prediction interval and 95% confidence interval

```
Fit        95% CI              95% PI
54.2418  (44.0487, 65.4948)  (36.5459, 75.4202)
```

***- though the value lies within the prediction interval it is due to the transformation that this median is varying. Thus it more important to see that the prediction intervals are satisfying or not for a MODEL TO BE DEEMED FIT.**

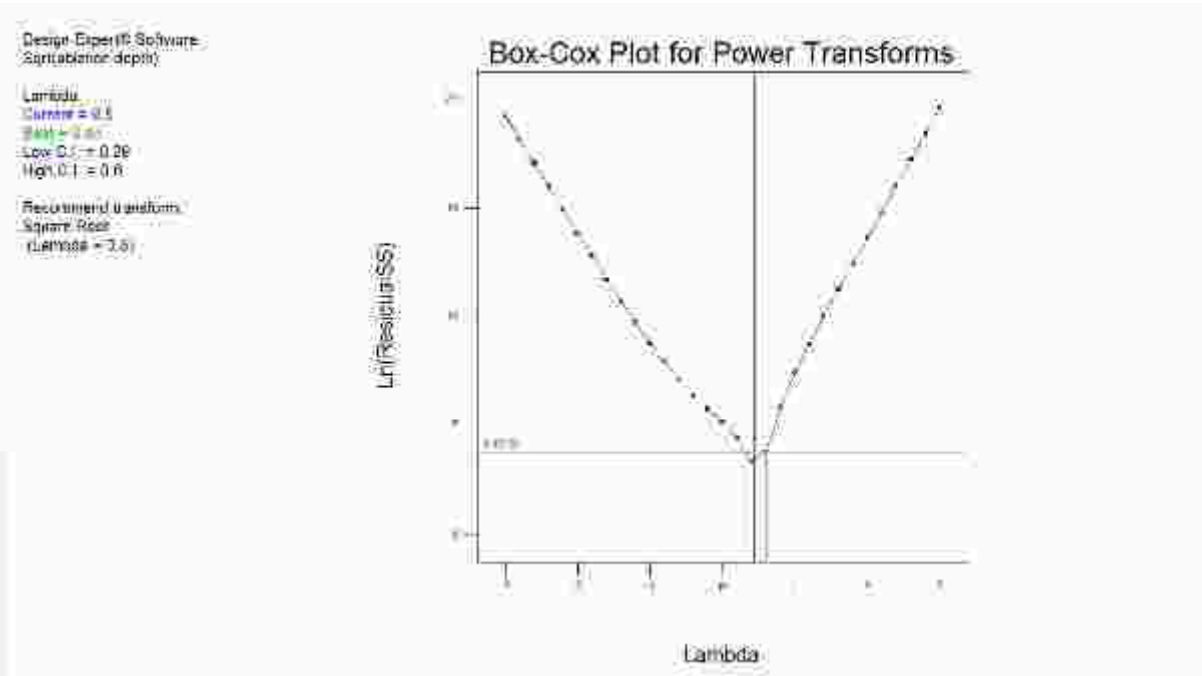
The problem however with this model is that we know there is a curvature but we do not know which variable it belongs and the linear model fitting unable to tell us. Thus a response surface is conducted with 8 additional axial face points which would enable to find out that factor. I am not aware how MINITAB is able to figure what value is MINITAB putting for the CT point in the regression equation. But it is a significant value. A face centered response was done since I could augment my existing design and since these were the maximum limits in the design space, CCD would not be possible. Thus I am sacrificing the rotatability since my aim is to just find the square term. Further studies can henceforth improve the model.

Ablation depth^0.5 = -3.05 + 0.00336 Pulse duration + 0.0593 Avg power + 0.0622 Overlap+ 0.00949 Freq + 0.001332 Avg power*Overlap - 0.000215 Avg power*Freq- 0.000190 Overlap*Freq -0.833 Ct Pt

How does minitab calculate the value for the CT pt in the regression eqn. Is it the average of all the center points of the response function??

RSM study on the Ablation depth

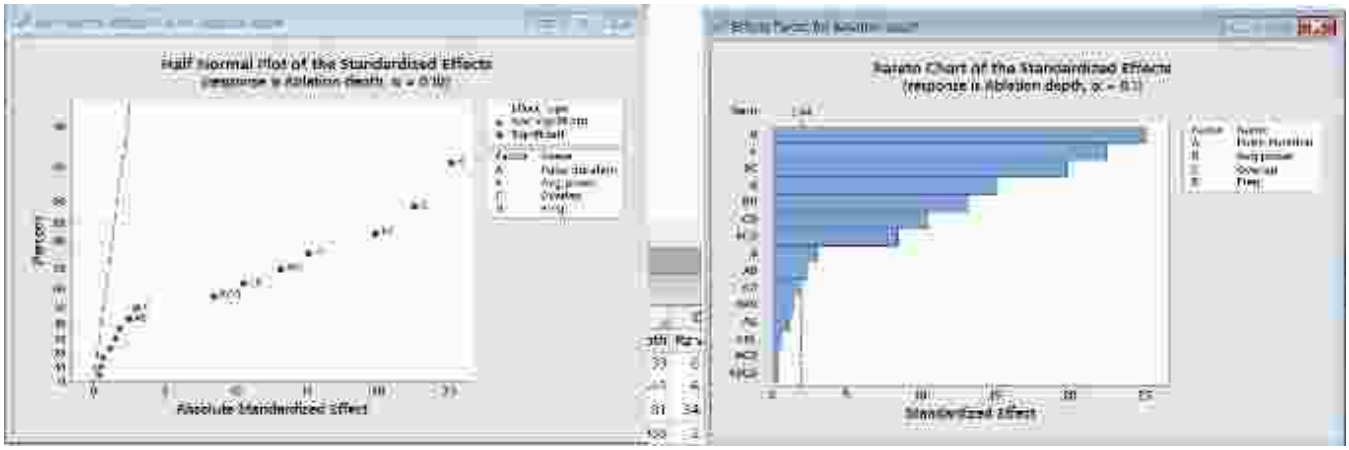
I already know that a square root model is fitting my data better. Incase this was not known the software “DESIGN EXPERT” enables you to analyse what transformation to use based on BOX-COX transformation check of the Ln(residuals)



Now performing RSM analysis using MINITAB without having any previous knowledge of this transformation but only including the most significant terms found out of the **first exercise (Pg 4).**

ie the significant factors were identified as A,B,C,D

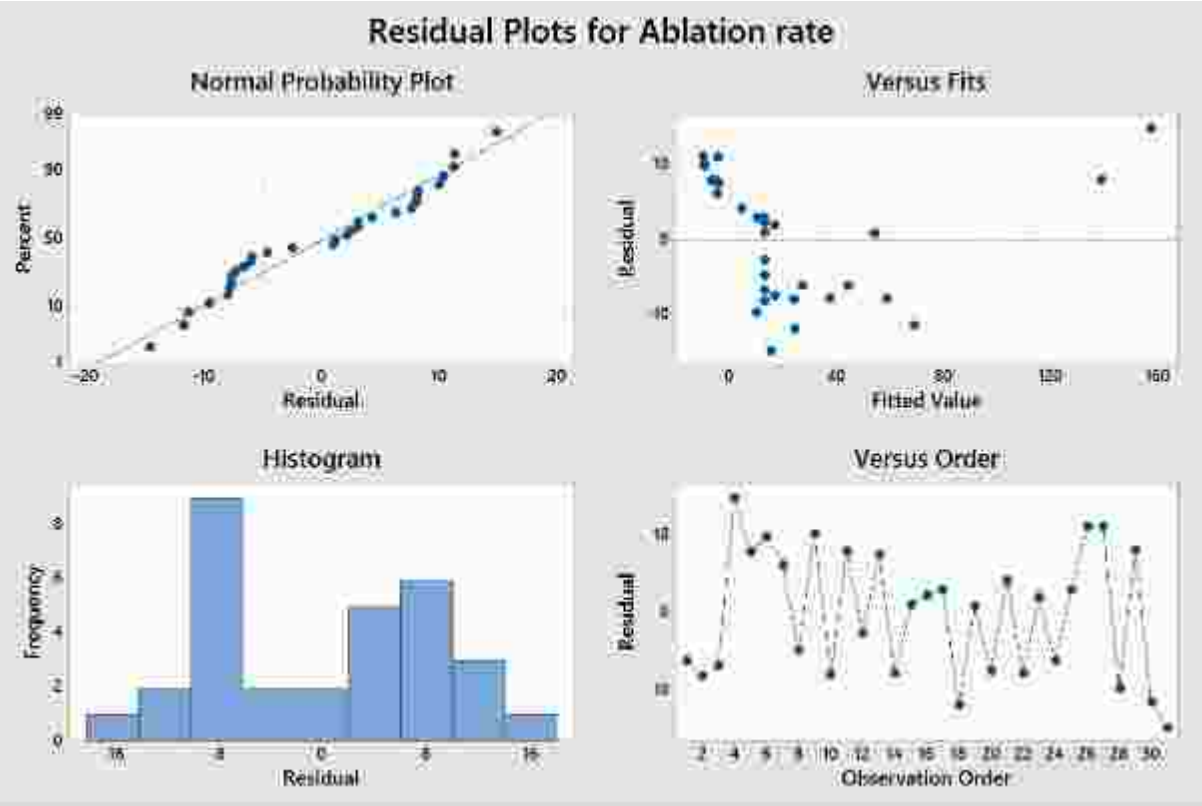
STEP 3: RSM ANALYSIS WITH QUADRATIC FUNCTION CONSISITING OF ALL TERMS



The design space is shown here. You can observe that the (-1) points are the only extra runs being conducted for the response surface.

	StdOrder	RunOrder	PtType	Blocks	Pulse duration	Avg power	Overlap	Freq	Ablation rate	Rz value
1	23	1	-1	1	104	60	50	200	21.261	9.6255
2	31	2	0	1	104	60	50	300	5.139	6.7325
3	30	3	0	1	104	60	50	300	6.543	6.5886
4	8	4	1	1	200	100	75	200	172.181	34.7602
5	2	5	1	1	200	20	25	200	1.058	2.9709
6	1	6	1	1	8	20	25	200	0.305	2.8620
7	13	7	1	1	8	20	75	400	1.630	2.8769
8	29	8	0	1	104	60	50	300	8.506	7.3554
9	11	9	1	1	8	100	25	400	0.838	21.8105
10	3	10	1	1	8	100	25	200	16.388	37.9083
11	14	11	1	1	200	20	75	400	3.090	3.4436
12	27	12	0	1	104	60	50	300	10.642	9.8387
13	12	13	1	1	200	100	25	400	3.497	24.9796
14	4	14	1	1	200	100	25	200	29.928	33.7215
15	28	15	0	1	104	60	50	300	14.307	9.3729
16	25	16	0	1	104	60	50	300	15.569	11.7102
17	17	17	-1	1	8	60	50	300	13.393	11.7490
18	6	18	1	1	200	20	75	200	12.818	13.4966
19	22	19	-1	1	104	60	75	300	55.127	12.6226
20	5	20	1	1	8	20	75	200	9.578	4.4498
21	21	21	-1	1	104	60	25	300	8.682	22.6681
22	15	22	1	1	8	100	75	400	50.949	18.9417
23	18	23	-1	1	200	60	50	300	18.955	13.2934
24	20	24	-1	1	104	100	50	300	38.129	25.4433
25	26	25	0	1	104	60	50	300	16.216	10.4101
26	19	26	-1	1	104	20	50	300	1.486	2.8083
27	24	27	-1	1	104	60	50	400	6.823	7.5482
28	10	28	1	1	200	20	25	400	0.679	2.7754
29	7	29	1	1	8	100	75	200	147.061	30.3521
30	16	30	1	1	200	100	75	400	57.720	18.5460
31	9	31	1	1	8	20	25	400	0.883	4.0989

Residuals check



We can observe the histogram is not being followed, also Residual vs fitted is showing a a kind of curve pattern. Thus there is a problem of fitting a square model directly to the ablation rate which calls for a transformation. Let us additionally look at the ANOVA

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	14	45079.0	3219.9	25.42	0.000
Linear	4	28910.4	7227.6	57.05	0.000
Pulse duration	1	192.7	192.7	1.52	0.235
Avg power	1	13076.8	13076.8	103.23	0.000
Overlap	1	11145.1	11145.1	87.98	0.000
Freq	1	4495.7	4495.7	35.49	0.000
Square	4	2547.3	636.8	5.03	0.008
Pulse duration*Pulse duration	1	0.5	0.5	0.00	0.953
Avg power*Avg power	1	42.7	42.7	0.34	0.570
Overlap*Overlap	1	677.1	677.1	5.35	0.034
Freq*Freq	1	7.6	7.6	0.06	0.810
2-Way Interaction	6	13621.3	2270.2	17.92	0.000
Pulse duration*Avg power	1	114.7	114.7	0.91	0.355
Pulse duration*Overlap	1	24.6	24.6	0.19	0.665
Pulse duration*Freq	1	63.9	63.9	0.50	0.488
Avg power*Overlap	1	7791.2	7791.2	61.50	0.000
Avg power*Freq	1	3453.8	3453.8	27.26	0.000
Overlap*Freq	1	2173.2	2173.2	17.15	0.001
Error	16	2026.9	126.7		
Lack-of-Fit	10	1907.3	190.7	9.57	0.006
Pure Error	6	119.6	19.9		
Total	30	47105.9			

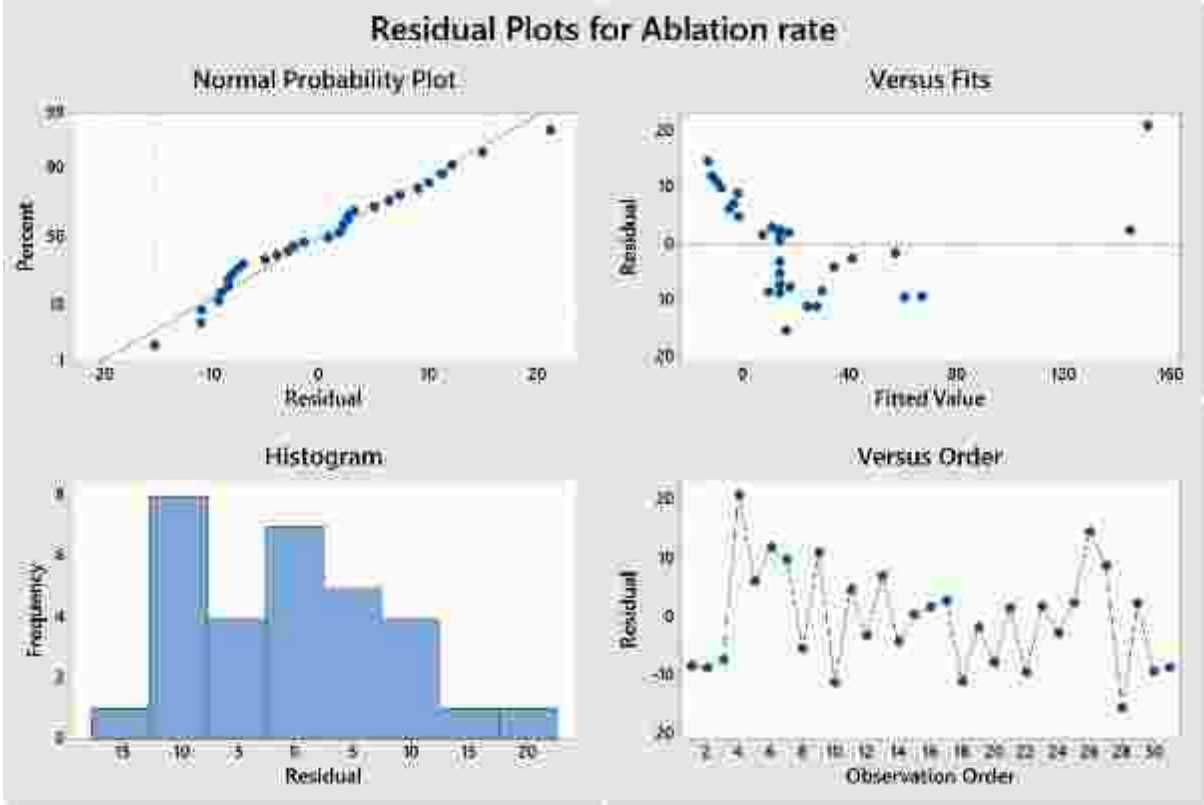
Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
11.2552	95.70%	91.93%	69.16%

We can see a lack of fit. So let us remove the non significant term (ie p>0.05) and remodel. Now another important this to note is that when you remove the terms the hierarchy of the variables are maintained. For example we see that pulse duration is not significant but it has significant interaction effects. Thus we should not eliminate pulse duration in the final eqn. However we have found out the square term that is playing an important role which is overlap.

So the model is refitted after removing the unwanted terms.

STEP 4: RE ANALYSIS OF RSM UTILIZING TERMS P<0.1 SO THAT MODEL MIGHT IMPROVE



Residual vs fitted after eliminating non significant terms. We see the normality is almost there but the residual vs fitted is still not random enough indicating that a transformation is required still. The ANOVA analysis shows the following

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	8	44825.5	5603.2	54.06	0.000
Linear	4	28910.4	7227.6	69.73	0.000
Pulse duration	1	192.7	192.7	1.86	0.186
Avg power	1	13076.8	13076.8	126.16	0.000
Overlap	1	11145.1	11145.1	107.52	0.000
Freq	1	4495.7	4495.7	43.37	0.000
Square	1	2497.0	2497.0	24.09	0.000
Overlap*Overlap	1	2497.0	2497.0	24.09	0.000
2-Way Interaction	3	13418.1	4472.7	43.15	0.000
Avg power*Overlap	1	7791.2	7791.2	75.16	0.000
Avg power*Freq	1	3453.8	3453.8	33.32	0.000
Overlap*Freq	1	2173.2	2173.2	20.97	0.000
Error	22	2280.4	103.7		
Lack-of-Fit	16	2160.9	135.1	6.78	0.013
Pure Error	6	119.6	19.9		
Total	30	47105.9			

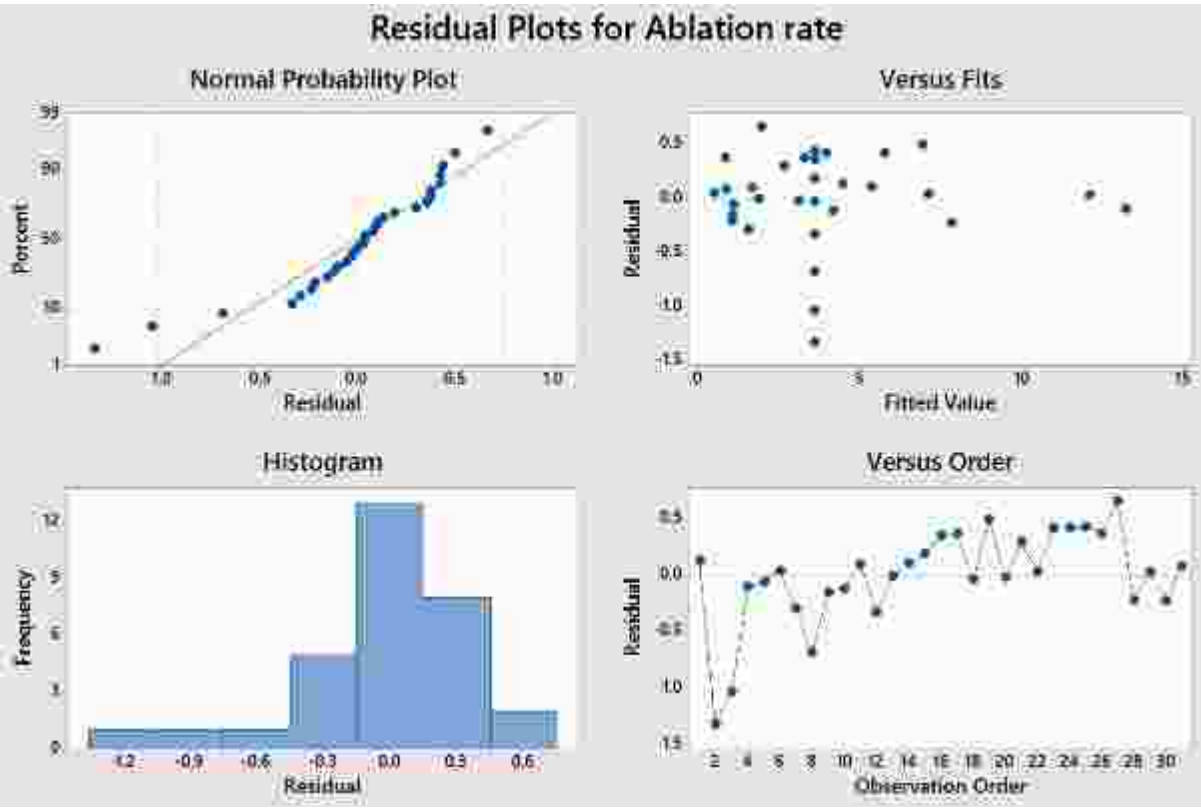
Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
10.1812	95.16%	93.40%	85.71%

Though the prediction value has increased, when we look at the term “lack of fit” since it is less than alpha 0.05, we can say the model does not accurately fit the data. So this is another important indicator that a transformation could help.

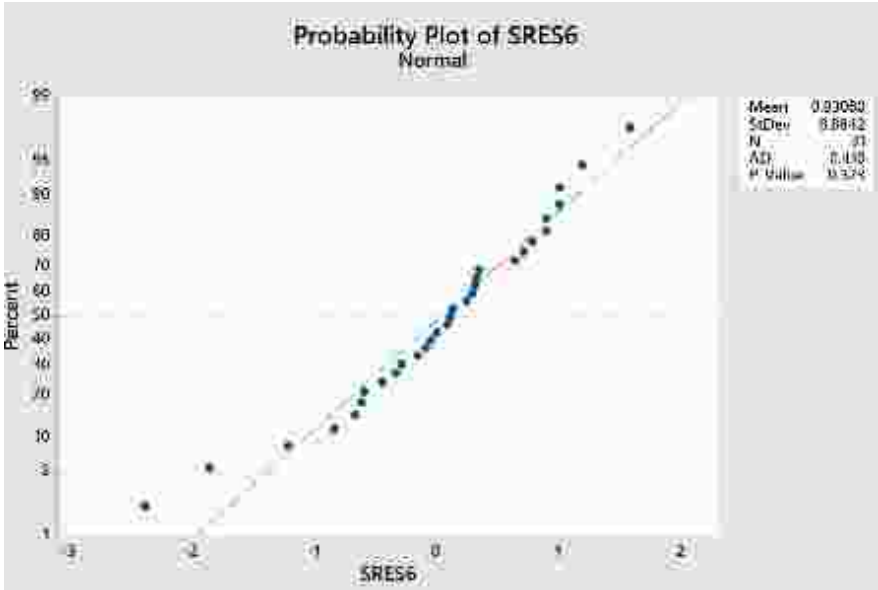
STEP 6: TRANSFORM RESPONSE TO SQR ROOT AND TRYING TO FIT QUADRATIC FUNCTION

So now we proceed ahead with a square root transformation of the and try fitting the data again with a quadratic equation.



Residuals vs fitted data show better random data. A normality test can be conducted to check for normality, but more important than that is to check if the histogram represents a normal figure which it does. Let us proceed to check the ANOVA and see if we can improve the model.

For the normal probability plot a normality test was conducted and found to be clear using the ANDERSON DARLING test. (results shown below)

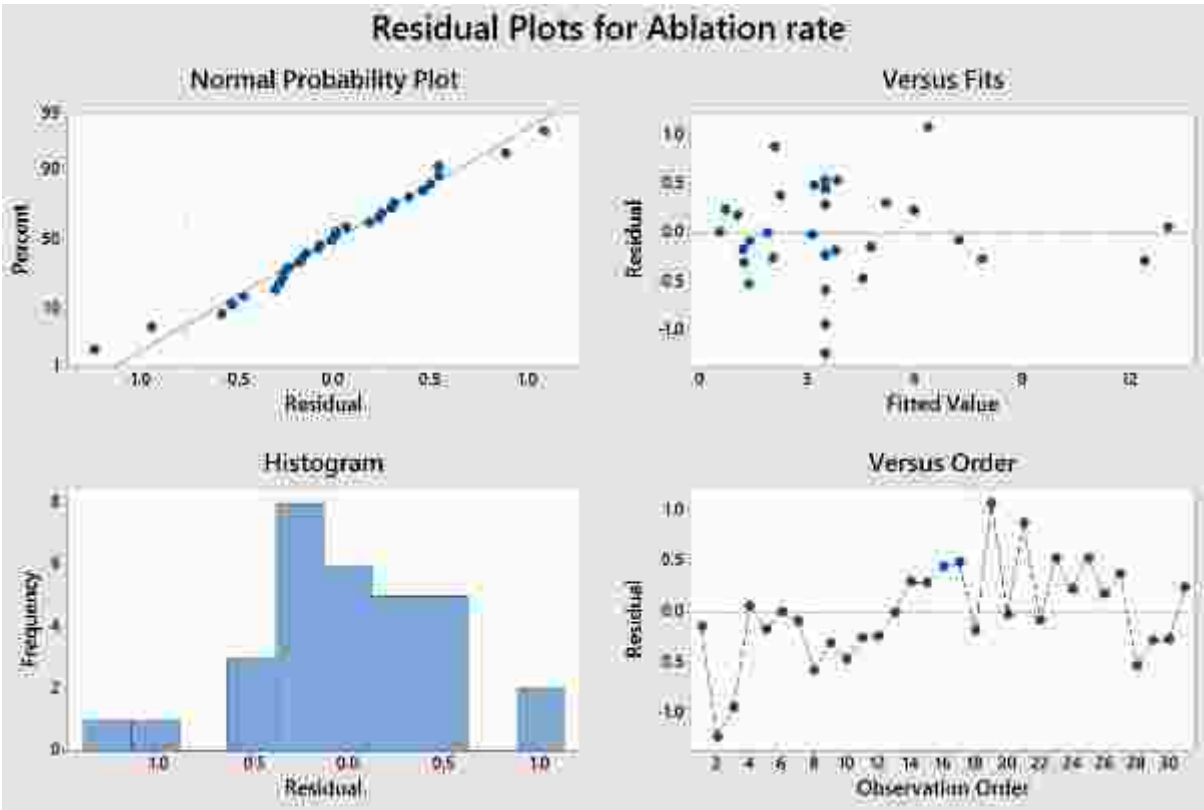



```
Box-Cox transformation  λ = 0.5


Analysis of Variance for Transformed Response

Source                              DF   Adj SS   Adj MS  F-Value  P-Value
Model                               14  270.863   19.347    57.06    0.000
  Linear                             4  221.467   55.367   163.30    0.000
    Pulse duration                   1    1.903    1.903     5.61    0.031
    Avg power                        1  108.483  108.483   319.96    0.000
    Overlap                          1   82.444   82.444   243.16    0.000
    Freq                             1   28.636   28.636    84.46    0.000
  Square                             4    4.946    1.236     3.65    0.027
    Pulse duration*Pulse duration    1    0.001    0.001     0.00    0.963
    Avg power*Avg power              1    0.223    0.223     0.66    0.430
    Overlap*Overlap                  1    3.711    3.711    10.95    0.004
    Freq*Freq                        1    0.371    0.371     1.10    0.311
  2-Way Interaction                  6   44.451    7.408    21.85    0.000
    Pulse duration*Avg power         1    0.392    0.392     1.16    0.298
    Pulse duration*Overlap           1    0.006    0.006     0.02    0.893
    Pulse duration*Freq              1    0.160    0.160     0.47    0.502
    Avg power*Overlap                1   28.396   28.396    83.75    0.000
    Avg power*Freq                   1   11.884   11.884    35.05    0.000
    Overlap*Freq                     1    3.613    3.613    10.66    0.005
Error                               16    5.425    0.339
  Lack-of-Fit                       10    2.497    0.250     0.51    0.833
  Pure Error                         6    2.928    0.488
Total                               30  276.288


Model Summary for Transformed Response

       S    R-sq  R-sq(adj)  R-sq(pred)
0.582286  98.04%     96.32%      95.65%
```

Now you can see a dramatic increase in the R sq values and prediction values. The lack of fit tells us that we cannot reject the null hypothesis that the model fits the data. However maybe

we can try improving the prediction value by removing the non significant terms as done earlier and we can check again the R sq values.

STEP 7: TRYING TO IMPROVE MODEL BY REMOVING P>0.1 TERMS



Here we can notice the histogram is more or less normal and other graphs seems to be conforming. Let us check the ANOVA analysis once again.

Box-Cox transformation λ = 0.5

Analysis of Variance for Transformed Response

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	8	269.217	33.652	104.70	0.000
Linear	4	221.467	55.367	172.26	0.000
Pulse duration	1	1.903	1.903	5.92	0.024
Avg power	1	108.483	108.483	337.51	0.000
Overlap	1	82.444	82.444	256.50	0.000
Freq	1	28.636	28.636	89.09	0.000
Square	1	3.858	3.858	12.00	0.002
Overlap*Overlap	1	3.858	3.858	12.00	0.002
2-Way Interaction	3	43.892	14.631	45.52	0.000
Avg power*Overlap	1	28.396	28.396	88.34	0.000
Avg power*Freq	1	11.884	11.884	36.97	0.000
Overlap*Freq	1	3.613	3.613	11.24	0.003
Error	22	7.071	0.321		
Lack-of-Fit	16	4.143	0.259	0.53	0.854
Pure Error	6	2.928	0.488		
Total	30	276.288			

Model Summary for Transformed Response

S	R-sq	R-sq(adj)	R-sq(pred)
0.566937	97.44%	96.51%	96.03%

There is no significant increase in prediction value. But this regression eqn is more simpler.

Coded Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	3.491	0.157	22.20	0.000	
Pulse duration	0.325	0.134	2.43	0.024	1.00
Avg power	2.455	0.134	18.37	0.000	1.00
Overlap	2.140	0.134	16.02	0.000	1.00
Freq	-1.261	0.134	-9.44	0.000	1.00
Overlap*Overlap	0.715	0.206	3.46	0.002	1.00
Avg power*Overlap	1.332	0.142	9.40	0.000	1.00
Avg power*Freq	-0.862	0.142	-6.08	0.000	1.00
Overlap*Freq	-0.475	0.142	-3.35	0.003	1.00

Regression Equation in Uncoded Units

```
sqrt ablation = -0.91 + 0.00339 Pulse duration + 0.0594 Avg power
                - 0.0517 Overlap + 0.00982 Freq + 0.001144 Overlap*Overlap
                + 0.001332 Avg power*Overlap - 0.000215 Avg power*Freq
                - 0.000190 Overlap*Freq
```

Fits and Diagnostics for Unusual Observations

Obs	sqrt ablation	Fit	Resid	Std Resid	
2	2.267	3.491	-1.225	-2.25	R
19	7.425	6.346	1.078	2.02	R

R Large residual

WOULD THE RESIDUAL BEING LEFT AS IT IS CREATE MUCH OF A PROBLEM? I remeasured it thrice, so I am quite sure it is not a measurement error. Can I leave it aside as some kind of random error that my model can't incorporate?

THIS IS THE FINAL EQN AND WE SEE THAT OVERLAP IS THE MAJOR SIGNIFICANT SQUARE TERM. We can simplify the equation by utilizing the ANOVA anlysis used after elimination of the terms, however there is no requirement for that now. We will now again check the predicted results. What is important to understand that a reasonable model has been generated with good accuracy utilizing minimal effort. A CCD can be done further to this to attain a more reliable model since it is a rotatable design. It is has better prediction capabilities and also finding the optimal is more suited using CCD method or Box Behnken design.

We will now check the predicted results of the confirmatory runs using the PREDICT function in MINITAB. This is different from the response optimizer which has to be utilized for verification runs.

Run details

A. Trial 1 – Actual reading - **<u>70.1848 [200ns 60 power 75 overlap 200 freq]</u>**

Predicted values with 95% prediction interval and 95% confidence interval

```
Fit        95% CI              95% PI
70.6967  (60.5402, 81.6405)  (50.0623, 94.8832)
```

B. Trial 2 – Actual reading – **<u>6.00873 [8ns 60 power 25overlap 200 freq]</u>**

Predicted values with 95% prediction interval and 95% confidence interval

```
   Fit        95% CI              95% PI
6.38645  (3.60912, 9.95099)  (1.42677, 14.8981)
```

C. Trial 3 – Actual reading – **<u>36.75709 * [100ns 100 power 75 overlap 400 freq]</u>**

Predicted values with 95% prediction interval and 95% confidence interval

```
Fit        95% CI              95% PI
56.7807  (46.0050, 68.6891)  (37.6910, 79.7679)
```

***- though the value lies within the prediction interval it is due to the transformation that this median is varying. Thus it more important to see that the prediction intervals are satisfying or not for a MODEL TO BE DEEMED FIT after transformation.**